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This report deals with the optimization of imperfect stiffened cylinders under destabilizing loads. The developed methodology is demonstrated through numerous examples. The research was performed under Grant AFOSR 74-2655 during the period 1 Feb 77 through 31 Jan 78. The report also contains a summary of the work performed under grant AFOSR 74-2655 during the period 1 Feb 74 - 31 Jan 78.

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THE EFFECT OF INITIAL IMPERFECTIONS ON OPTIMAL STIFFENED CYLINDERS UNDER TORSION*

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ABSTRACT

The nonlinear buckling analysis of geometrically imperfect, thin circular, cylindrical, stiffened shells under pure torsion and torsion combined with axial compression, for various transverse and in-plane boundary conditions, is first performed. A methodology is presented for predicting critical conditions (limit point loads) for such configurations. This methodology is based on the smeared technique (closely spaced stiffeners), the von Kármán-Donnell nonlinear kinematic relations in the presence of initial imperfections, and linearly elastic behavior. The computational procedure employs a Fourier series type of separated solution and by employing the Galerkin procedure in the circumferential direction and identities of trigonometric functions the field equations are reduced to a system of ordinary differential equations. These equations are then solved by the finite difference scheme. Numerical results for numerous stiffened and unstiffened configurations are presented. Some of these configurations are used as benchmarks for the developed methodology, since numerical solutions for them have been reported in the open literature.

Then, the effect of initial geometric imperfections on optimal stiffened configurations is assessed. This is accomplished by computing the critical load at the optimum design point as well as at design points in the surrounding space for a given geometric imperfection. The optimum point was obtained through the use of linear buckling analysis. The comparison shows (in this case) that the optimum point for torsion-loaded perfect stiffened cylinders is also the optimum design point for same geometry imperfect stiffened cylinders provided that the proper factor is used with the applied torsion to account for imperfection sensitivity. Finally, a summary of the work performed under this grant is presented at the end of this report.

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NOTATIONS

A	Area
A_i, B_i	Fourier coefficients (radial displacement expression)
A_i^o, B_i^o	Fourier coefficient (imperfection expression)
A_x, A_y	Stringer and ring cross-sectional area
C_i, D_i	Fourier coefficients (stress function expression)
D	Flexural stiffness of the skin
E	Young's modulus of elasticity
E_{xx_p}	Extensional stiffness of the skin
e_x, e_y	Stringer and ring eccentricities (positive inward)
e_{Av}, γ_{Av}	Average unit end shortening, unit end twist
F	Stress function
I_{xc}, I_{yc}	Stringer and ring moment of inertia about their centroidal axes.
K	Number of terms in truncated Fourier Series
ℓ_x, ℓ_y	Stringer and ring spacings
ℓ	Mesh point
L	Total length of the shell
M_{xx}, M_{yy}, M_{xy}	Moment resultants
m	Number of axial half waves
N_{xx}, N_{yy}, N_{xy}	Stress resultants
$\bar{N}_{xx}, \bar{N}_{xy}$	Applied compressive load and applied torsion load
$\bar{N}_{xx_{c\ell}}, \bar{N}_{xy_{c\ell}}, P_{c\ell}$	Classical buckling loads
$\bar{N}_{xx_{cr}}, \bar{N}_{xy_{cr}}, P_{cr}$	Critical loads (limit point)

n	Number of circumferential full waves
NP	Number of points in axial direction
p	Pressure
p_i^1, p_i^2	Fourier coefficient (pressure expression)
R	Radius of the cylinder
t	Skin thickness
U_T	Total Potential
u, v	In-plane displacements
w	Radial displacement (positive inward)
w^o	Radial geometric imperfection
x, y, z	Coordinate system
Z	Batdorf curvature parameter $[= L^2(1-\nu^2)^{1/2}/R.t]$
$\epsilon_{xx}, \epsilon_{yy}, \epsilon_{xy}$	Reference surface strains
$\varkappa_{xx}, \varkappa_{yy}, \varkappa_{xy}$	Reference surface changes in curvature and torsion
$\bar{\lambda}_{xx}, \bar{\lambda}_{yy}$	Smeared extensional stiffnesses of stringers and rings
$\bar{\rho}_{xx}, \bar{\rho}_{yy}$	Smeared flexural stiffnesses of stringers and rings
ν	Poisson's ratio
$(\ )' = [\ ]_{,x}$	Derivative with respect to x
Λ	Knockdown factor (for individual load application $\Lambda = N_{cr}/N_{c\ell}$)

1. INTRODUCTION

Most of the investigations reported in the open literature (see Ref. 1 and the cited references therein for a complete and comprehensive historical sketch) which consider buckling of imperfect shell configurations, employ an isotropic constant thickness geometry and a uniform axial compression. Very few consider other constructions and even fewer torsional loading. Loo[2], in 1954, reported the results of his investigation of the effect of initial imperfections on the critical condition for simply supported, constant thickness, isotropic, thin, cylindrical shell loaded in torsion. His formulation contains numerous approximations and simplifying assumptions. Nash[3] removed some of the simplifying assumptions and extended Loo's work to the case of clamped boundary conditions. Both of these investigations have made important contributions to the state of the art, but their reported results can only be considered as qualitative.

A different approach, based on Koiter's initial postbuckling theory (Ref. 4 and 5), was employed by Budiansky[6] in dealing with the same problem as Loo and Nash. Results were presented for classical simply supported and two sets of clamped boundary conditions.

The present report is an extension of the work reported by the authors in Refs. 1, 7. These references present the buckling analysis of imperfect, stiffened, thin cylinders of finite length under uniform axial compression and/or lateral pressure, for axisymmetric or at most symmetric imperfections and various boundary conditions. The present work removes the limitation on the imperfection shape and considers individual or combined application of uniform axial compression, lateral pressure (which can

be position dependent), and uniform end torsion. The methodology of Ref. 1 is modified in order to accomodate the general shape of the imperfection and the new load conditions. Results of this investigation are presented both in graphical and tabular form for several examples.

In addition, the optimization of imperfect stiffened cylinders under torsion is investigated. This is accomplished by first finding the optimum geometry of the corresponding perfect configuration by using linear buckling analysis and a safety factor to account for imperfection sensitivity, and then by assessing the effect of geometric imperfections on this optimum geometry and the surrounding design configurations.

2. MATHEMATICAL FORMULATION

Given an imperfect (w^o), stiffened, thin, circular cylindrical shell of finite length, L, and various boundary conditions under the application of axial compression, lateral pressure and torsion, the equilibrium equations can be derived by employing the steps outlined in Ref. 1. By introducing the Airy stress function as

$$
\begin{aligned}
N_{xx} &= -\bar{N}_{xx} + F,_{yy} \\
N_{yy} &= F,_{xx} \\
N_{xy} &= \bar{N}_{xy} - F,_{xy}
\end{aligned}
\tag{1}
$$

where $\bar{N}_{xx}$ is the applied uniform compression and $\bar{N}_{xy}$ is the applied torsional stress resultant, the equilibrium and compatibility equations in terms of w and F become

$$
\begin{aligned}
DL_h[w] - Lq[F] - \frac{F,_{yy}}{R} - L[F, w+w^o] + \bar{N}_{xx}\,(w,_{xx}+w^o,_{xx}) - 2\bar{N}_{xy}\,(w,_{xy}+w^o,_{xy}) \\
- p = 0
\end{aligned}
\tag{2}
$$

$$
L_d[F] + L_q[w] + \frac{1}{2} L\,[w, w+2w^o] + \frac{w,_{xx}}{R} = 0 \tag{3}
$$

where the operators L_h, L_q and L_d are defined in Ref. 1, and p is the applied pressure.

The expression for the total potential is given by

$$
\begin{aligned}
U_T = \frac{1}{2E_{xx_p}} \int_A \left[\beta_1 F,_{yy}^2 + \beta_2 F,_{xx}^2 + \beta_3 F,_{xx} F,_{yy} + \beta_4 F,_{xy}^2\right] dA \\
+ \frac{D}{2} \int_A \left[\alpha_1 w,_{yy}^2 + \alpha_2 w,_{xx}^2 + \alpha_3 w,_{xx} w,_{yy} + \alpha_4 w,_{xy}^2\right] dA
\end{aligned}
$$

$$- \frac{\bar{N}_{xx}}{2E_{xx_p}} \int_A \left[2\beta_1 F,_{yy} + \beta_3 F_{xx} \right] dA - \frac{\bar{N}_{xy}}{2E_{xx_p}} \int_A 2\beta_4 F,_{xy} \, dA$$

$$- \int_A pwdA + \frac{\pi RL}{E_{xx_p}} \left(\beta_1 \bar{N}_{xx}^2 + \beta_4 \bar{N}_{xy}^2 \right) - 2\pi RL \left(e_{AV} \bar{N}_{xx} + \gamma_{AV} \bar{N}_{xy} \right) \tag{4}$$

where the coefficients α_i and β_i are defined in Ref. 1, and the symbols e_{AV} and γ_{AV} denote the average end shortening and average shear strain respectively. The mathematical expressions for these quantities are given by

$$e_{AV} = a_1 \bar{N}_{xx} - \frac{1}{2\pi RL} \int_0^{2\pi R} \int_0^{L} \left[a_1 F,_{yy} + a_2 F,_{xx} + a_3 w,_{xx} \right.$$

$$\left. + a_4 w,_{yy} - \frac{1}{2} w,_x \left(w,_x + 2w^o,_x \right) \right] dxdy \tag{5}$$

$$\gamma_{AV} = \frac{2\bar{N}_{xy}}{(1-\nu)E_{xx_p}} - \frac{1}{2\pi RL} \int_0^{2\pi R} \int_0^{L} \left[\frac{2F,_{xy}}{(1-\nu)E_{xx_p}} + w,_x w,_y + w,_y w^o,_x + w,_x w^o,_y \right] dxdy \tag{6}$$

Similarly, the expressions for the end shortening and shear strain at y = 0 are given by

$$e(y{=}0) = a_1 \bar{N}_{xx} - \frac{1}{L} \int_0^{L} \left[a_1 F,_{yy} + a_2 F,_{xx} + a_3 w,_{xx} + a_4 w,_{yy} \right.$$

$$\left. - \frac{1}{2} w,_x \left(w,_x + 2w^o,_x \right) \right]_{y=0} dx \tag{5a}$$

$$\gamma(y{=}0) = \frac{2\bar{N}_{xy}}{(1-\nu)E_{xx_p}} - \frac{1}{L} \int_0^{L} \left[\frac{2F,_{xy}}{(1-\nu)E_{xx_p}} + w,_x w,_y + w^o,_x w,_y + w,_x w^o,_y \right]_{y=0} dx \tag{6a}$$

The boundary conditions are developed in a manner similar to Ref. 1, and the general computer program is written so as to accomodate any combination of transverse and in-plane boundary conditions (SS_i, CC_i, FF_i, i = 1,2,3,4)

$$
\begin{array}{lll}
SS & w = M_{xx} = 0 & \quad 1. \; F,_{xy} = F,_{yy} = 0 \\
CC & w = w,_x = 0 & \quad 2. \; F,_{xy} = 0;\; u = C \\
FF & Q_x^* = M_{xx} = 0 & \quad 3. \; v = C;\; F,_{yy} = 0 \\
 & & \quad 4. \; v = C;\; u = C
\end{array}
\tag{7}
$$

where C is a constant, and the conditions in u and v may be expressed in terms of w and F as in Ref. 9.

The first step in the methodology, employed herein as well as in Ref. 1, is to reduce the governing equations, Eqs.(2) and (3) from a system of coupled nonlinear partial differential equations to a system of coupled nonlinear ordinary differential equations. This is accomplished by employing the following separated form for w and F.

$$
\begin{aligned}
w(x,y) &= \sum_{i=0}^{K} \left[A_i(x) \cos \frac{iny}{R} + B_i(x) \sin \frac{iny}{R} \right] \\
F(x,y) &= \sum_{i=0}^{2K} \left[C_i(x) \cos \frac{iny}{R} + D_i(x) \sin \frac{iny}{R} \right]
\end{aligned}
\tag{8}
$$

In addition, if one considers the imperfection to be, in general, asymmetric and the applied pressure position dependent, then similar expressions may be employed for $w^o(x,y)$ and $p(x,y)$,

$$w^o(x,y) = \sum_{i=0}^{K} \left[A_i^o(x) \cos \frac{iny}{R} + B_i^o(x) \sin \frac{iny}{R} \right]$$

$$p(x,y) = \sum_{i=0}^{K} \left[p_i^1(x) \cos \frac{iny}{R} + p_i^2(x) \sin \frac{iny}{R} \right] \tag{9}$$

The reduction to ordinary differential equations is accomplished through the following steps.

(1) First, Eqs. (8) and (9) are substituted into the compatibility equation, Eq. (3). Then, by employing trigonometric identities involving products and the linear independence of the sine and cosine terms, (4K +1) coupled nonlinear ordinary differential equations are obtained. This substitution clearly shows why the summation in the F(x,y) expression is from zero to 2K. These equations, which relate the $A_i(x)$, $B_i(x)$, $C_i(x)$, etc., are:

<u>for i = 0</u>

$$C_o'' = \frac{1}{d_{11}} \left\{ -q_{11} A_o'' - \frac{A_o}{R} + \frac{n^2}{4R^2} \sum_{j=1}^{K} j^2 \left[\left(A_j + 2A_j^o \right) A_j + \left(B_j + 2B_j^o \right) B_j \right] \right\} \tag{10}$$

<u>for i = 1,2,3,...2K</u>

(α) from the cosine terms

$$d_{11} C_i'''' - 2d_{12} \left(\frac{in}{R}\right)^2 C_i'' + d_{22} \left(\frac{in}{R}\right)^4 C_i + \delta_i \left[q_{11} A_i'''' - 2q_{12} \left(\frac{in}{R}\right)^2 A_i'' \right.$$

$$\left. + q_{22} \left(\frac{in}{R}\right)^4 A_i \right] + \delta_i A_i''/R - \frac{\delta_i}{2} \left(\frac{in}{R}\right)^2 \left(A_i + 2A_i^o \right) A_o''$$

$$- \left(\frac{n}{2R}\right)^2 \sum_{j=1}^{K} \left\{ \left[(i+j)^2 \delta_{i+j} \left(A_{i+j} + 2A_{i+j}^o \right) + \left(2 - \eta_{j-i}^2 \right) (i-j)^2 \delta_{|i-j|} \times \right. \right.$$

$$\left(A_{|i-j|} + 2A^{o}_{|i-j|}\right)\Big]A_j'' + \Big[(i+j)^2\delta_{i+j}\left(B_{i+j} + 2B^{o}_{i+j}\right)$$

$$- \eta_{i-j}(i-j)^2\delta_{|i-j|}\left(B_{|i-j|} + 2B^{o}_{|i-j|}\right)\Big] B_j'' + 2\Big[(i+j)\delta_{i+j}\left(A_{i+j}' + 2A^{o\,\prime}_{i+j}\right)$$

$$- \eta_{i-j}\,|i-j|\,\delta_{|i-j|}\left(A_{|i-j|}' + 2A^{o\,\prime}_{|i-j|}\right)\Big]j\,A_j'$$

$$+ 2\Big[(i+j)\delta_{i+j}\left(B_{i+j}' + 2B^{o\,\prime}_{i+j}\right) + \left(2 - \eta^2_{j-1}\right)|i-j|\delta_{|i-j|}\,x$$

$$\left(B_{|i-j|}' + 2B^{o\,\prime}_{|i-j|}\right)\Big]\,jB_j' + \Big[\delta_{i+j}\left(A_{i+j}'' + 2A^{o\,\prime\prime}_{i+j}\right)$$

$$+ \left(2 - \eta^2_{j-i}\right)\delta_{|i-j|}\left(A_{|i-j|}'' + 2A^{o\,\prime\prime}_{|i-j|}\right)\Big]j^2A_j + \Big[\delta_{i+j}\left(B_{i+j}'' + 2B^{o}_{i+j}\right)$$

$$- \eta_{i-j}\delta_{|i-j|}\left(B_{|i-j|}'' + 2B^{o\,\prime\prime}_{|i-j|}\right)\Big]\,j^2B_j\Big\} = 0 \qquad (11)$$

(β) from the sine terms

$$d_{11}D_i'''' - 2d_{12}\left(\frac{in}{R}\right)^2 D_i'' + d_{22}\left(\frac{in}{R}\right)^4 D_i + \delta_i\Big[q_{11}B_i'''' - 2q_{12}\left(\frac{in}{R}\right) B_i''$$

$$+ q_{22}\left(\frac{in}{R}\right)^4 B_i\Big] + \delta_i B_i''/R - \frac{\delta_i}{2}\left(\frac{in}{R}\right)^2\left(B_i + 2B_i^{o}\right) A_o''$$

$$- \left(\frac{n}{2R}\right)^2 \sum_{j=1}^{K} \Big\{\Big[(i+j)^2\delta_{i+j}\left(B_{i+j} + 2B^{o}_{i+j}\right) + \eta_{i-j}(i-j)^2\delta_{|i-j|}\,x$$

$$\left(B_{|i-j|} + 2B^{o}_{|i-j|}\right)\Big] A_j'' + \Big[-(i+j)^2\delta_{i+j}\left(A_{i+j} + 2A^{o}_{i+j}\right)$$

$$+ \left(2 - \eta^2_{j-i}\right)(i-j)^2 \delta_{|i-j|} \left(A_{|i-j|} + 2A^o_{|i-j|}\right)\Big] B''_j$$

$$-2\Big[-(i+j)\delta_{i+j}\left(B'_{i+j} + 2B^{o\prime}_{i+j}\right) + \left(2 - \eta^2_{j-i}\right)|i-j|\delta_{|i-j|} \times$$

$$\left(B'_{|i-j|} + 2B^{o\prime}_{|i-j|}\right)\Big] jA'_j - 2\Big[(i+j)\delta_{i+j}\left(A'_{i+j} + 2A^{o\prime}_{i+j}\right)$$

$$+ \eta_{i-j}|i-j|\delta_{|i-j|}\left(A'_{|i-j|} + 2A^{o\prime}_{|i-j|}\right)\Big] jB'_j + \Big[\delta_{i+j}\left(B''_{i+j} + 2B^o_{i+j}\right)$$

$$+ \eta_{i-j}\delta_{|i-j|}\left(B''_{|i-j|} + 2B^{o\prime\prime}_{|i-j|}\right)\Big] j^2 A_j + \Big[-\delta_{i+j}\left(A''_{i+j} + 2A^{o\prime\prime}_{i+j}\right)$$

$$+ \left(2 - \eta^2_{j-i}\right)\delta_{|j-i|}\left(A''_{|i-j|} + 2A^{o\prime\prime}_{|i-j|}\right)\Big] j^2 B_j = 0 \qquad (12)$$

where the prime denotes differentiation with respect to x and

$$\delta_\ell = \begin{cases} 0 & \ell > K \\ 1 & \ell \le K \end{cases} \quad ; \quad \eta_\ell = \begin{cases} -1 & \ell < 0 \\ 0 & \ell = 0 \\ 1 & \ell > 0 \end{cases} \qquad (13)$$

(2) Second, Eqs. (8) and (9) are substituted into the equilibrium equation, Eq. (2), and the error is made orthogonal to cos $\frac{iny}{R}$ and sin $\frac{iny}{R}$ for i = 0,1,2...K. This is a Galerkin-type procedure with respect to the circumferential direction. The vanishing of the (2k+1) Galerkin integrals leads to the following system of (2k+1) nonlinear ordinary differential equations (equilibrium)

for $i = 0$

$$A_o''''\left(Dh_{11} + q_{11}^2/d_{11}\right) + A_o''\left(\frac{2q_{11}}{Rd_{11}}\right) + A_o\left(\frac{1}{R^2 d_{11}}\right) + \bar{N}_{xx}\left(A_o'' + A_o''\right)$$

$$- p_o^1 - \left(\frac{n}{2R}\right)^2 \sum_{j=1}^{K} j^2 \left\{\frac{q_{11}}{d_{11}}\left[\left(A_j + 2A_j^o\right) A_j'' + \left(A_j'' + 2A_j^{o''}\right) A_j\right.\right.$$

$$+ 2\left(A_j' + 2A_j^{o'}\right) A_j' + \left(B_j + 2B_j^o\right) B_j'' + \left(B_j'' + 2B_j^{o''}\right)B_j + 2\left(B_j' + 2B_j^{o'}\right)B_j'\Big]$$

$$+ \frac{1}{Rd_{11}}\left[\left(A_j + 2A_j^o\right) A_j + \left(B_j + 2B_j^o\right)B_j\right] - 2\left[\left(A_j + A_j^o\right) C_j'' + \left(B_j + B_j^o\right) D_j''\right.$$

$$\left.\left. + 2\left(A_j' + A_j^{o'}\right) C_j' + 2\left(B_j' + B_j^{o'}\right) D_j' + \left(A_j'' + A_j^{o''}\right) C_j + \left(B_j'' + B_j^{o''}\right)D_j\right]\right\} = 0 \tag{14}$$

for $i = 1,2,\ldots K$

(α) when the weighting function is $\cos \frac{iny}{R}$

$$D\left[h_{11}A_i'''' - 2h_{12}\left(\frac{in}{R}\right)^2 A_i'' + h_2\left(\frac{in}{R}\right)^4 A_i\right] + \bar{N}_{xx}\left(A_i'' + A_i^{o''}\right)$$

$$- 2\bar{N}_{xy}\left(B_i' + B_i^{o'}\right)\left(\frac{in}{R}\right) - \left[q_{11}C_i'''' - 2q_{12}\left(\frac{in}{R}\right)^2 C_i'' + q_{22}^2\left(\frac{in}{R}\right)^4 C_i\right]$$

$$- C_i''/R - p_i^1 + \left(\frac{in}{R}\right)^2 \frac{(A_i + A_i^o)}{d_{11}}\left\{-q_{11}A_o'' - A_o/R + \left(\frac{n}{2R}\right)^2 \sum_{j=1}^{K} j^2\left[\left(A_j + 2A_j^o\right)A_j\right.\right.$$

$$\left.\left. + \left(B_j + 2B_j^o\right) B_j\right]\right\} + \frac{1}{2}\left(\frac{n}{R}\right)^2 \sum_{j=1}^{2K} \left\{\left[(i+j)^2 \delta_{i+j}\left(A_{i+j} + A_{i+j}^o\right)\right.\right.$$

$$+ \left(2 - \eta_{j-i}^2\right)(i-j)^2\delta_{|i-j|}\left(A_{|i-j|} + A^o_{|i-j|}\right)\Big] C_j'' + \Big[(i+j)^2\delta_{i+j} \times$$

$$\left(B_{i+j} + B^o_{i+j}\right) - \eta_{i-j}(i-j)^2\delta_{|i-j|}\left(B^o_{|i-j|} + B^o_{|i-j|}\right)\Big] D_j''$$

$$+ 2\Big[(i+j)\delta_{i+j}\left(A'_{i+j} + A^{o'}_{i+j}\right) - \eta_{i-j}|i-j|\delta_{|i-j|}\left(A'_{|i-j|} + A^{o'}_{|i-j|}\right)\Big] jC_j'$$

$$+ 2\Big[(i+j)\delta_{i+j}\left(B'_{i+j} + B^{o'}_{i+j}\right) + \left(2 - \eta_{j-i}^2\right)|i-j|\delta_{|i-j|}\left(B'_{|i-j|} + B^{o'}_{|i-j|}\right)\Big] jD_j'$$

$$+ \Big[\delta_{i+j}\left(A''_{i+j} + A^{o''}_{i+j}\right) + \left(2 - \eta_{j-i}^2\right)\delta_{|i-j|}\left(A''_{|i-j|} + A^{o''}_{|i-j|}\right)\Big] j^2 C_j$$

$$+ \Big[\delta_{i+j}\left(B''_{i+j} + B^{o''}_{i+j}\right) - \eta_{i-j}\delta_{|i-j|}\left(B''_{|i-j|} + B^{o''}_{|i-j|}\right)\Big] j^2 D_j\Big\} = 0 \quad (15)$$

(β) when the weighting function is $\sin \frac{iny}{R}$

$$D\left[h_{11}B_i'''' - 2h_{12}\left(\frac{in}{R}\right)^2 B_i'' + h_{22}\left(\frac{in}{R}\right)^4 B_i\right] + \bar{N}_{xx}\left(B_i'' + B_i^{o''}\right)$$

$$+ 2\bar{N}_{xy}\left(A_i' + A_i^{o'}\right)\left(\frac{in}{R}\right) - \left[q_{11}D_i'''' - 2q_{12}\left(\frac{in}{R}\right)^2 D_i'' + q_{22}\left(\frac{in}{R}\right)^4 D_i\right] - \frac{D_i''}{R}$$

$$- p_i^2 + \left(\frac{in}{R}\right)^2 \frac{(B_i + B_i^o)}{d_{11}}\left\{-q_{11}A_o'' - \frac{A_o}{R} + \left(\frac{n}{2R}\right)^2 \sum_{j=1}^{K} j^2\Big[\left(A_j + 2A_j^o\right)A_j\right.$$

$$\left. + \left(B_j + 2B_j^o\right)B_j\Big]\right\} + \frac{1}{2}\left(\frac{n}{R}\right)^2 \sum_{j=1}^{2K}\Big\{\Big[(i+j)^2\delta_{i+j}\left(B_{i+j} + B^o_{i+j}\right)$$

$$+ \eta_{i-j}(i-j)^2\delta_{|i-j|}\left(B_{|i-j|} + B^o_{|i-j|}\right)\Big] C''_j + \Big[-(i+j)^2\delta_{i+j}\left(A_{i+j} + A^o_{i+j}\right)$$

$$+ \left(2 - \eta^2_{j-i}\right)(i-j)^2\delta_{|i-j|}\left(A_{|i-j|} + A^o_{|i-j|}\right)\Big] D''_j - 2\Big[-(i+j)\delta_{i+j} \times$$

$$\left(B'_{i+j} + B^o_{i+j}\right) + \left(2 - \eta^2_{j-i}\right)|i-j|\delta_{|i-j|}\left(B'_{|i-j|} + B^{o'}_{|i-j|}\right)\Big] jC'_j$$

$$- 2\Big[(i+j)\delta_{i+j}\left(A'_{i+j} + A^o_{i+j}\right) + \eta_{i-j}|i-j|\delta_{|i-j|}\left(A'_{|i-j|} + A^{o'}_{|i-j|}\right)\Big] jD'_j$$

$$+ \Big[\delta_{i+j}\left(B''_{i+j} + B^{o''}_{i+j}\right) + \eta_{i-j}\delta_{|i-j|}\left(B''_{|i-j|} + B^{o''}_{|i-j|}\right)\Big] j^2C_j$$

$$+ \Big[-\delta_{i+j}\left(A''_{i+j} + A^{o''}_{i+j}\right) + \left(2 - \eta^2_{j-i}\right)\delta_{|i-j|}\left(A''_{|i-j|} + A^{o''}_{|i-j|}\right)\Big] j^2D_j\Big\} = 0$$

(16)

Note that the equations governing the response of the imperfect configuration to any level of the applied loading ($\bar{N}_{xx}, \bar{N}_{xy}$, p^1_i and p^2_i) for a given shape and magnitude of the imperfection (A^o_i, B^o_i) are reduced to a system of (6k+2) equations, Eqs. (10), (11), (12), (14), (15) and (16), in (6k+2) unknowns, A_i(i=0,1,..K), B_i(i=1,2...k), C_i(i=0,1,...2k), and D_i(i=1,2...k). Note that C_o through Eq. (1), has been eliminated from the remaining equations. In these governing equations, one more undetermined parameter is present, the wave number, n. This number is established by requiring the total potential to be a minimum at the critical condition (see Refs. 1 and 9). Because of this, the expression for the total potential i needed, which is given below

$$U_T = \frac{\pi R}{E_{xx_p}} \int_0^L \left\{ \frac{B_2}{(d_{11})^2} \left[-q_{11} A_o'' - \frac{A_o}{R} + \left(\frac{n}{2R}\right)^2 \sum_{j=1}^{K} j^2 \left\langle \left(A_j + 2A_j^o\right) A_j \right. \right. \right.$$

$$\left. \left. + \left(B_j + 2B_j^o\right) B_j \right\rangle \right]^2 + \frac{1}{2} \sum_{j=1}^{2K} \left[\beta_1 \left(\frac{jn}{R}\right)^4 \left(C_j^2 + D_j^2\right) + \beta_2 \left(C_j'' + D_j''\right) \right.$$

$$\left. \left. - \beta_3 \left(\frac{jn}{R}\right)^2 \left(C_j C_j'' + D_j D_j''\right) + \beta_4 \left(\frac{jn}{R}\right)^2 \left(C_j'^2 + D_j'^2\right) \right] \right\} dx + \pi R D \int_0^L \left\{ \alpha_2 (A_o'')^2 \right.$$

$$+ \frac{1}{2} \sum_{j=1}^{K} \left[\alpha_1 \left(\frac{jn}{R}\right)^4 \left(A_j^2 + B_j^2\right) + \alpha_2 \left\{ \left(A_j''\right)^2 + \left(B_j''\right)^2 \right\} - \alpha_3 \left(\frac{jn}{R}\right)^2 x \right.$$

$$\left. \left. \left(A_j A_j'' + B_j B_j''\right) + \alpha_4 \left(\frac{jn}{R}\right)^2 \left\{ \left(A_j'\right)^2 + \left(B_j'\right)^2 \right\} \right] \right\} dx$$

$$- \frac{\pi R \beta_3 \bar{N}_{xx}}{d_{11} E_{xx_p}} \int_0^L \left\{ -q_{11} A_o'' - \frac{A_o}{R} + \left(\frac{n}{2R}\right)^2 \sum_{j=1}^{K} j^2 \left[\left(A_j + 2A_j^o\right) A_j \right. \right.$$

$$\left. \left. + \left(B_j + 2B_j^o\right) B_j \right] \right\} dx - \pi R \int_0^L \left\{ 2p_o^1 A_o + \sum_{j=1}^{K} \left[p_j^1 A_j + p_j^2 B_j \right] \right\} dx$$

$$+ \pi R L \left[\frac{1}{E_{xx_p}} \left(\beta_1 \bar{N}_{xx}^2 + \beta_4 \bar{N}_{xy}^2 \right) - 2 \left(e_{AV} \bar{N}_{xx} + \gamma_{AV} \bar{N}_{xy} \right) \right] \tag{17}$$

In addition, the expressions for the average end shortening and shear strain [see Eqs. (5) and (6)] become

$$e_{AV} = a_1\bar{N}_{xx} + \frac{1}{L}\int_0^L \left\langle \frac{a_2}{d_{11}} \left\{ q_{11}A_o'' + \frac{A_o}{R} - \left(\frac{n}{2R}\right)^2 \sum_{j=1}^{K} j^2 \left[\left(A_j + 2A_j^o\right) A_j \right.\right.\right.$$

$$\left.\left. + \left(B_j + 2B_j^o\right) B_j\right]\right\} - a_3A_o'' + \frac{1}{2} A_o'\left(A_o' + 2A_o^{o'}\right) + \frac{1}{4} \sum_{j=1}^{K} \left[A_j'\left(A_j' + 2A_j^{o'}\right)\right.$$

$$\left. \left. + B_j'\left(B_j' + 2B_j^{o'}\right)\right] \right\rangle dx \qquad (18)$$

$$\gamma_{AV} = \frac{2\bar{N}_{xy}}{(1-\nu)E_{xx_p}} - \frac{1}{2L}\int_0^L \sum_{j=1}^{K} \left(\frac{jn}{R}\right) \left[A_j'\left(B_j + B_j^o\right) + A_j^{o'}B_j\right.$$

$$\left. - B_j'\left(A_j + A_j^o\right) - A_jB_j^{o'}\right] dx \qquad (19)$$

$$e_{AV}(y=0) = a_1\bar{N}_{xx} + \frac{1}{L}\int_0^L \left\langle \frac{a_2}{d_{11}} \left\{ q_{11}A_o'' + \frac{A_o}{R} - \left(\frac{n}{2R}\right)^2 \sum_{j=1}^{K} j^2 \left[A_j\left(A_j + 2A_j^o\right)\right.\right.\right.$$

$$\left.\left. + B_j\left(B_j + 2B_j^o\right)\right]\right\} - a_3A_o'' + \sum_{j=1}^{2K} \left[a_1\left(\frac{jn}{R}\right)^2 C_j - a_2C_j''\right]$$

$$\left. + \sum_{j=1}^{K} \left[a_4\left(\frac{jn}{R}\right)^2 A_j - a_3A_j''\right] + \frac{1}{2}\left[\sum_{i=0}^{K} A_i'\right]\left[\sum_{j=0}^{K}\left(A_j' + 2A_j^{o'}\right)\right] \right\rangle dx \qquad (20)$$

$$\gamma_{AV}(y=0) = \frac{2\bar{N}_{xy}}{(1-\nu)E_{xx_p}} - \frac{1}{L}\int_0^L \left\langle \frac{2}{(1-\nu)E_{xx_p}} \sum_{j=1}^{2K} \left(\frac{jn}{R}\right) D_j' \right.$$

$$\left. + \left[\sum_{i=0}^{K} A_i'\right]\left[\sum_{j=1}^{K}\left(\frac{jn}{R}\right)\left(B_j + B_j^o\right)\right] + \left[\sum_{i=1}^{K}\left(\frac{in}{R}\right) B_i\right]\left[\sum_{j=0}^{K} A_j^{o'}\right] \right\rangle dx \qquad (21)$$

Note that, in the total potential expression, Eq. (17), one must use Eqs. (18) and (19) for e_{AV} and γ_{AV}; but Eqs. (20) and (21) are listed here because in plotting load versus some characteristic displacement or rotation, they are used for these characteristic parameters. The reason for their use instead of that of the average values, Eqs. (18) and (19), is that better and more distinguishable plots are thus possible.

Finally, the appropriate boundary conditions are also expressed in terms of A_i, B_i, C_i and D_i.

3. SOLUTION PROCEDURE

The solution procedure employed herein is a modification of the procedure described in Ref. 1. A generalization of Newton's method, applicable to differential equations, serves to reduce the nonlinear field equations, Eqs. (11), (12), (14), (15), and (16), and the appropriate boundary conditions to a sequence of linear systems. In this method, the iteration equations are derived by assuming that the solution is achieved by a small correction to an approximate solution (initially taken as the linear solution). These small corrections are obtained from the solution of the linearized (with respect to the corrections) differential equations. The ordinary differential equations are cast into the form of finite difference equations as in Ref. 1. The unknown vector Z contains (12K+2) elements

$$\{Z\} = \{A_o, A_1 \ldots A_K, B_1 \ldots B_K, C_1 \ldots C_{2K}, D_1 \ldots D_{2K},$$
$$A_o'', A_1'', \ldots A_K'', B_1'' \ldots B_K'', C_1'' \ldots C_{2K}'', D_1'' \ldots D_{2K}\}$$

Note that the second derivatives (of A_i, B_i, C_i and D_i) are considered as independent elements of the vector $\{Z\}$.

By using one fictitious point on each exterior side of the cylinder ends, one can write a system of (12K+2)(NP+2) difference equations, where NP denotes the number of mesh points. This system of difference equations can be solved by the special algorithm reported in Ref. 10, when a unique solution exists for a given set of applied loads. When the set of applied loads corresponds to a critical condition (limit point), a unique solution does not exist and thus the solution of the system of difference equations fails to converge. On the basis of these observations, the description of the solution procedure

is as follows: First, the system of difference equations is solved for a small level of the applied load. Then a multiple of this solution is used for a small increase in the load parameter. At each step the value of n is needed to accomplish a solution. To this end, different values of n are used to obtain a solution. The solution that corresponds to the value of n that minimizes the total potential is considered as the correct one for that step. Numerical integration is used to find the total potential. The number of n-values needed to be tried at each step is small, since the circumferential mode does not vary significantly with small increases in the applied load. For the purpose of minimizing the time required to accomplish a solution, judgement, based on experience, is used, which provides a balance between load step size (and consequently number of steps) and number of n-values at each step. Numerical integration is also used, at each step, to compute the corresponding unit end shortening and/or average shear strain. Finally, this procedure is continued until the solution of the system of difference equations fails to converge. The associated load level corresponds to the critical condition.

In the case of combined loads, the procedure is virtually the same, but only one of the loads is increased while the remaining are kept constant. The ones kept constant are those which are small by comparison to the linear theory individual load application critical condition. For example, consider the case of a configuration under the application of uniform axial compression, $\bar{N}_{xx}$, and torsion, $\bar{N}_{xy}$. For this case, suppose that one is interested in finding the effect that a given imperfection has on the critical condition (this means to find the critical curve in the $\bar{N}_{xx}$, $\bar{N}_{xy}$-space that corresponds

to this imperfection, and compare it to the critical curve that corresponds to perfect geometry and is obtained through linear theory). Through the use of linear theory the critical curve can be found for a given structural configuration (see Fig. 1). The intercepts denote critical loads under individual application (from the typical curve of Fig. 1 these values are: $\bar{N}_{xx_{cr}} = 5\bar{A}$ and $\bar{N}_{xy_{cr_L}} = 8\bar{A}$ where $\bar{A}$ is some numerical constant). From the nonlinear procedure, outlined herein for individual load application and a given imperfection, one can find the nonlinear theory critical loads (see Fig. 1, $\bar{N}_{xx_{cr_{NL}}} = 3\bar{A}$, and $\bar{N}_{xy_{cr_{NL}}} = 7\bar{A}$). In order to find the complete curve one can (a) fix $\bar{N}_{xy} = \bar{A}$ and $2\bar{A}$ and employ the methodology by increasing $\bar{N}_{xx}$ to find points I and II and (b) fix $\bar{N}_{xx} = \bar{A}$ and increase $\bar{N}_{xy}$ to find point III.

There is an alternate approach in constructing the nonlinear critical curve. This approach requires that both $\bar{N}_{xx}$ and $\bar{N}_{xy}$ be increased linearly (along line OP corresponding to some angle θ) in order to find the critical combination, point III, for that θ value. Then vary θ from zero to $\pi/2$ and the complete curve is generated.

Both approaches are incorporated into the computer program (Appendix A).

Regardless of the approach used, the ratio $|\overline{OIII}| / |\overline{OP}|$ is a measure of the imperfection sensitivity (instead of $|\overline{QIII}| / |\overline{QP'}|$). This ratio may be called knockdown factor and denoted by Λ.

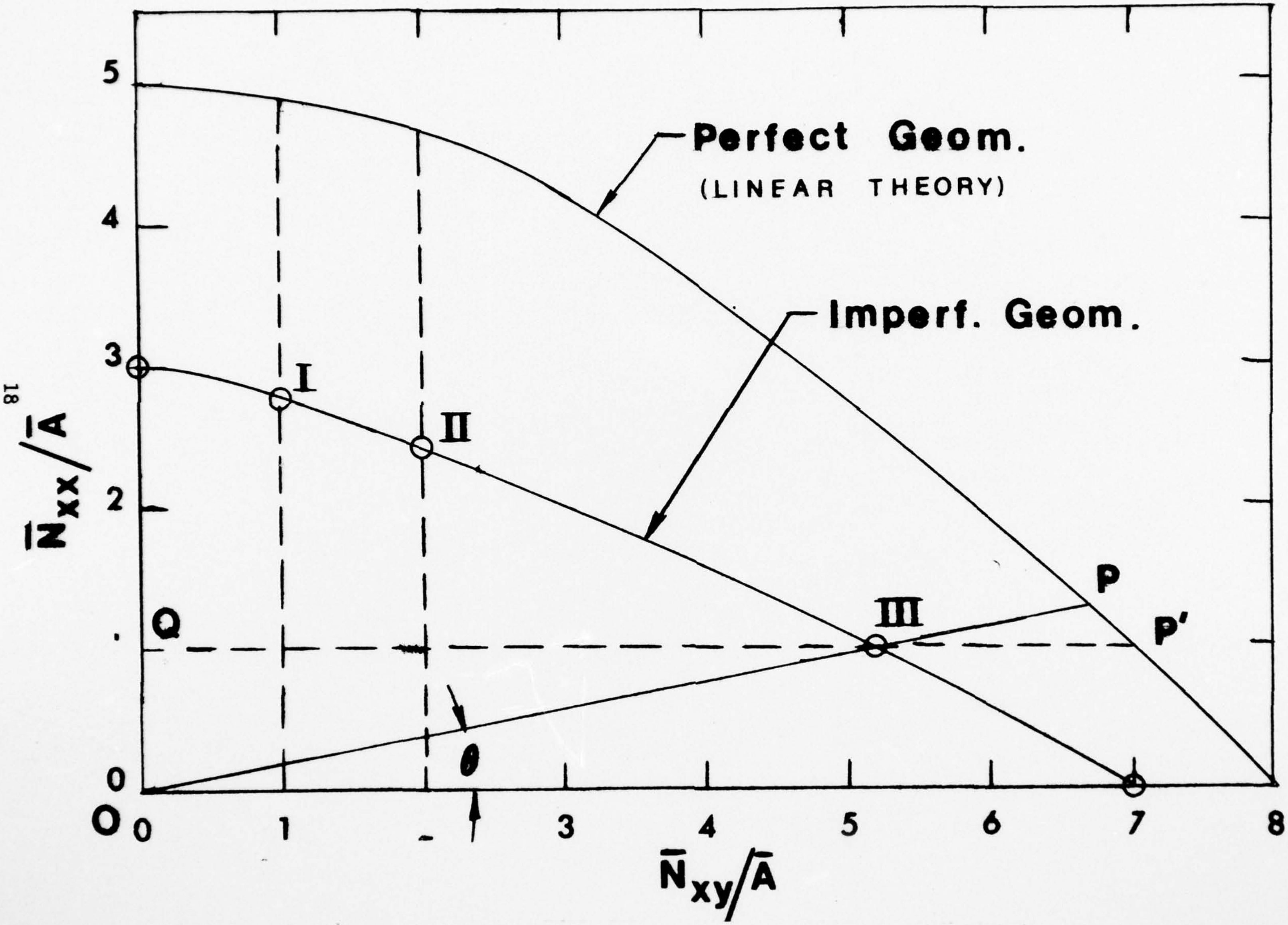


Fig. 1. Critical Conditions Under Combined Loads (Typical Qualitative Curves)

4. NUMERICAL RESULTS AND DISCUSSION

The present methodology is demonstrated through a number of illustrative examples. Numerical solutions are obtained by employing the Georgia Tech high speed digital computer CDC-CYBER 70, Model 74-28.

A general computer program is written, which includes the following desirable features (Appendix A):

(1) It is applicable to a stiffened configuration, in either or both directions, as well as to an unstiffened configurations.

(2) It accomodates all possible boundary conditions (SSi, CCi, FFi, etc.) and it can easily be modified to accomodate elastic end restraints.

(3) The number of Fourier terms, K, can be as large as needed for accuracy. The same is true for the number of mesh points, NP, in the finite difference scheme.

(4) The shape of the geometric imperfection is unrestricted.

(5) It is applicable to any individual or combined application of uniform axial compression, torsion, and space-dependent lateral pressure.

(6) The CPU time required to obtain a solution is reasonably small. For example, by using $K = 1$ and $NP = 57$ (826 unknowns) a solution (critical load and all intermediate steps) is obtained in 18 seconds. For low load levels a convergent solution is obtained through two iterations. For load levels, approaching the limit point, a convergent solution is obtained through six iterations. The solution has converged if the percent difference in response between two consecutive iterations is smaller than 10^{-4}.

The numerical results for all illustrative examples (ten) are presented in tabular form in Table 1. The geometric parameters and load conditions

Table 1: Final results for Imperfect Stiffened Cylindrical Shell

Example No.	$E = (10.5)10^6$ Geometric Parameters										Loading	Linear buckling load perfect cylinder				Imperfection Amplitude δ/t	Nonlinear Solution Imperfect Cylinder				
	L	R	t	Z	$\frac{\ell_x}{t}$	$\frac{\ell_y}{t}$	$\bar{\lambda}_{xx}$	$\bar{\lambda}_{yy}$	$\bar{\rho}_{xx}$	$\bar{\rho}_{yy}$		$\bar{N}_{xx_{c\ell}}$	$\bar{N}_{xy_{c\ell}}$	$p_{c\ell}$	$n_{c\ell}$		$\bar{N}_{xx_{cr}}$	$\bar{N}_{xy_{cr}}$	p_{cr}	Λ	n_{cr}
1	4.	4.	0.04	95.4	6.	3.	0.91	0.455	100.	20.	$\bar{N}_{xx}$	19790.	-	-	4	1.0	15250 15630	-	-	0.771 - 0.790	4
2	4.	4.	0.04	95.4	6.	6.	0.91	0.91	100.	100.	p	-	-	7060.	4	1.0	-	-	6500.	0.92	4
3	4.	4.	0.04	95.4	-6.	-6.	0.91	0.91	100.	100.	$\bar{N}_{xx} = \frac{pr}{2}$, p	16200.	-	8100.	4	1.0	9385.	-	4693.	0.579	4
4	4.	8.	0.1886	10.	0.	0.	0.	0.	0.	0.	$\bar{N}_{xy}$	-	30660.	-	8	1.0	-	16500.	-	0.538	8
5	16.	4.	0.006	10000.	0.	0.	0.	0.	0.	0.	$\bar{N}_{xy}$	-	7.04	-	9	1.0	-	5.0	-	0.714	9
6	4.	4.	0.04	95.4	6.	3.	0.91	0.455	100.	20.	$\bar{N}_{xy}$	-	26260.	-	5	1.0	-	21500.	-	0.819	5
7	4.	4.	0.04	95.4	6.	3.	0.91	0.455	100.	20.	$\bar{N}_{xy}$	-	26260.	-	5	2.0	-	20500.	-	0.781	4
8	4.	4.	0.04	95.4	6.	3.	0.91	0.455	100.	20.	$\bar{N}_{xy}$	-	26260.	-	5	3.0	-	20000.	-	0.762	4
9	4.	4.	0.04	95.4	6.	3.	0.91	0.455	100.	20.	$\bar{N}_{xy}$	-	26260.	-	5	5.0	-	20000.	-	0.762	4
10	4.	4.	0.04	95.4	6.	3.	0.91	0.455	100.	20.	$\bar{N}_{xx} = 8000$ $\bar{N}_{xy}$	8000.	20280.	-	4	1.0	8000.	16250	-	0.856	5

for each example number are shown on this table. In addition, the perfect geometry (linear theory) results, the imperfection analysis results (present nonlinear theory), and the associated knockdown factor, Λ, are shown. For all ten examples the boundary conditions are taken to be classical simply supported (SS3). Poisson's ratio for examples 4 and 5 is 0.3333 while for the remaining examples is 0.3.

Examples 1, 2, and 3 have been reported in Refs. 1 and 7. The imperfection considered, for these examples, herein is

$$w^o(x,y) = t \sin \frac{m\pi x}{L}\left(\cos \frac{ny}{R} + \sin \frac{ny}{R}\right) .$$

Although a more general imperfection shape is used in this paper (in Refs. 1 and 7 the imperfection shape was taken to be symmetric, $\cos \frac{ny}{R}$ only) the results are the same because, in the absence of torsion, there is no coupling between sine and cosine terms.

Examples 4 and 5 correspond to an unstiffened configuration under torsion only, and they correspond to cases that have been worked out previously (see Ref. 5) by employing a Koiter-type analysis. The difference between these examples is the curvature parameter ($Z = 10$ for Example 4, and $Z = 10^4$ for Example 5). A comparison between the present results and those of Ref. 6 shows very good agreement for both cases. The imperfection for these two examples was taken to be

$$w^o(x,y) = \delta \sum_{m=1}^{5} \left[A_m \cos \frac{ny}{R} + B_m \sin \frac{ny}{R}\right] \sin \frac{m\pi x}{L} \tag{22}$$

with $\delta = t$ and where A_m and B_m are the elements of the corresponding perfect geometry linear theory eigenvector.

Note that in the case of torsion there are two different eigenvectors (linear theory), one corresponding to positive torsion and one to negative torsion. Since the imperfection shape is considered to be similar to this eigenvector, the analysis is performed to check the effect of one on the other. It is found that an imperfection shape similar to the positive torsion eigenvector has, virtually, no effect on the load carrying capacity when the torsion is applied in the negative direction. (The shell is insensitive and thus $\Lambda = 1$). This reinforces the contention that the shell is sensitive to imperfection shapes which are similar to the corresponding perfect geometry linear theory eigenvector. This observation is made for all other examples with torsional loads.

Examples 6 through 9 correspond to a stiffened configuration (same for all four), imperfection shapes characterized by Eq. (22), and different imperfection amplitudes (δ = t, 2t, 3t, and 5t respectively). The results for these cases are also presented graphically on Figs. 2 and 3. Two observations are worth mentioning here. First, the "guessed" postbuckling behavior (see dotted line on Fig. 2) indicates that shells loaded in torsion are not as sensitive to geometric imperfections as those loaded in axial compression. This is in agreement with the qualitative results of Ref. 3. Second, stiffened configurations of the same curvature parameter (based on actual thickness) or a weighted curvature parameter (based on a weighted thickness that includes the smeared stiffener contribution) are not as sensitive as the corresponding unstiffened configurations. According to Ref. 6 (see Table 1; Case I, and Fig. 6 of this reference) the knockdown factor for an unstiffened configuration with $Z < 96$ is definitely smaller than 0.6 (Λ<0.6), while for these stiffened geometries $\Lambda \geq 0.86$.

Finally, Example 10 corresponds to the same stiffened configuration as Examples 1 and 6 through 9 but under combined application of axial compression and torsion. The imperfection shape, for this example also, is taken to be similar to the classical perfect geometry eigenvector. Since, for this geometry, the analysis is performed for individual load application (Examples 1 and 6), the knockdown factor is computed as described in the previous section (see Fig. 1).

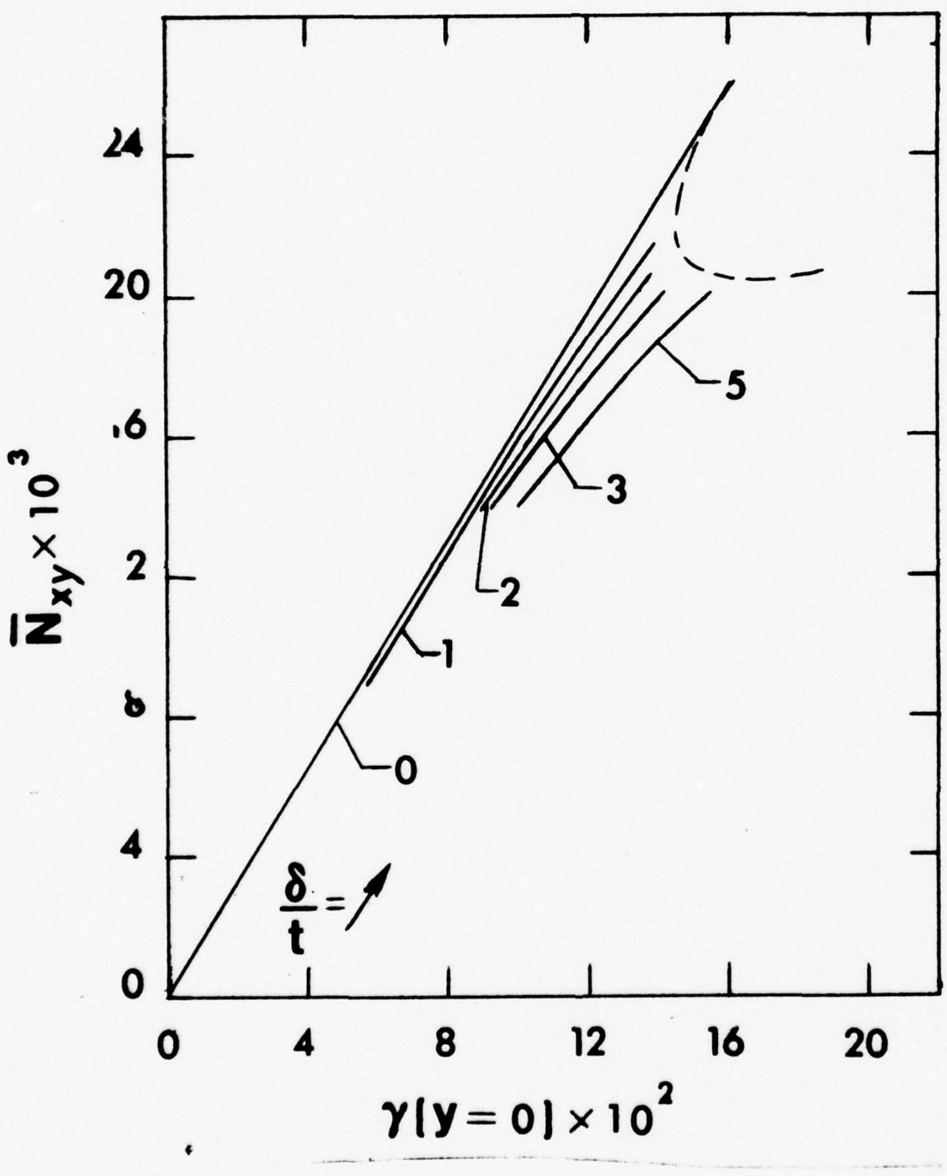


Fig. 2. Load Versus Angle of Twist for Imperfect Stiffened Cylinders (Examples 6-9).

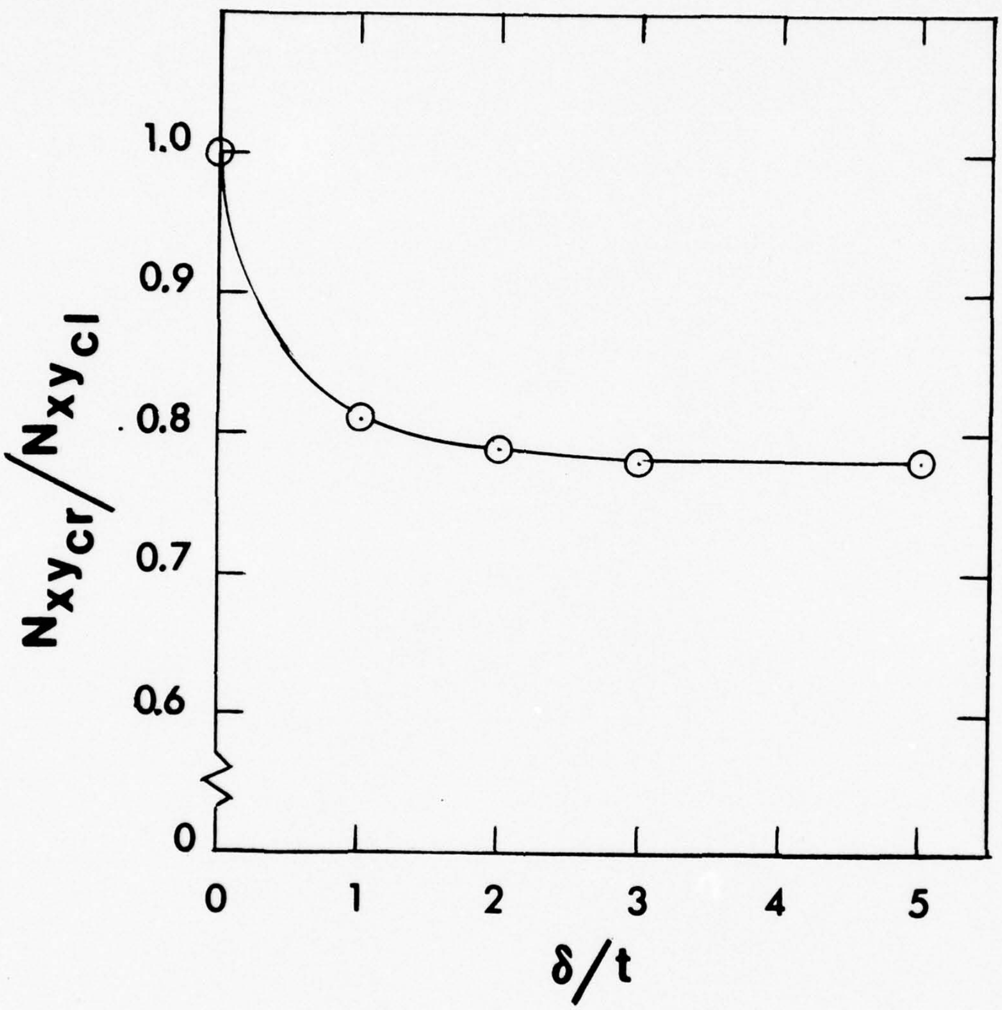


Fig. 3. Knockdown Factor Versus Imperfection Amplitude (Examples 6-9).

5. OPTIMAL CYLINDERS IN TORSION

Since no stiffened or unstiffened shell configuration is free of initial geometric imperfections, the optimization of such configurations requires the specification of both the amplitude and shape of the geometric imperfection and the use of a nonlinear buckling analysis in the solution methodology. Both requirements present difficulties of various degrees, the most serious of which is the inclusion of the nonlinear buckling analysis in the optimization algorithm. Because of this, an alternate approach will be employed. This approach is much simpler in terms of execution computer time and does lead to a reasonable final solution. Before presenting and demonstrating this alternate approach,let us state exactly the problem in hand: Given an internally stiffened, thin, circular, cylindrical, imperfect shell of specified material, radius, length, and imperfection (amplitude and shape) find the size, shape and spacings of the stiffeners, and the skin thickness such that it can safely carry a prescribed torsional load with minimum weight.

The alternate approach consist of the following steps:

1) Employ a safety factor with the applied load, which accounts, among other things, for the imperfection sensitivity of the stiffened shell.

2) Perform an optimization of the corresponding perfect geometry configuration by employing the methodology described and demonstrated in Ref. 11.

3) Using the specified imperfection, perform a nonlinear stability analysis (see chapters 2 through 4) on the optimum configuration (from step 2). Through this analysis find the corresponding knockdown factor. If it is close to the guessed number used in step 1 proceed with the

remaining steps, otherwise modify the safety factor appropriately and repeat steps one and two.

4) On the basis of step 2 the design space surrounding the optimum configuration is available. Choose design points in this space (their weight is higher than that of the optimum) and perform a nonlinear stability analysis in order to establish the corresponding knockdown factors. This comparison will establish the optimum design point in the presence of specified initial geometric imperfections.

The entire procedure is demonstrated herein by employing a design configuration of Ref. 11.

6.1 Design Example

The example used herein to demonstrate the optimization procedure is Example 1 of Ref. 11. The specified parameters are given below:

$$R = 85 \text{ in.} \;;\; L = 100 \text{ in.} \;;\; E_x = E_y = E = 10.5 \times 10^6 \text{psi.}$$

$$\nu = 0.33 \;;\; \rho_{sk} = \rho_x = \rho_y = 0.101 \text{ lbs/in.}^3$$

The applied torsion (stress resultant) is taken to be 300 lbs/in. and by assuming a safety factor of approximately 1.4 to account for imperfection sensitivity (knockdown factor = 0.715) the design applied torsion used in the linear buckling analysis optimization procedure (Ref. 11) is 418.5 lbs. In considering various types of stiffeners in Ref. 11, it is concluded that the best combination of shapes corresponds to T-stringers and rectangular rings (TS-RR).

The imperfection shape assumed for all configurations is taken to be similar to the linear theory buckling mode and the amplitude of imperfec-

tion is taken to be the same for all design points employed in the comparison ($\bar{\delta}$ = 0.1215 in.).

Note that the imperfection is characterized by

$$w^{o}(x,y) = \delta \sum_{m=0}^{7} \left(A_m \cos \frac{ny}{R} + B_m \sin \frac{ny}{R}\right) \sin \frac{m\pi x}{L}$$

where A_m and B_m are elements characterizing the linear theory buckling mode, each divided by the coefficient of the most influencing term. Through this the largest of A_m, B_m is equal to 1.0 and the remaining absolute values are less than 1.0. Thus, the maximum amplitude of the imperfection is given by

$$\bar{\delta} = \delta \left| \sum_{m=0}^{7} \left(A_m \cos \frac{ny}{R} + B_m \sin \frac{ny}{R}\right) \sin \frac{m\pi y}{L} \right|_{max}$$

For all configuration considered, δ is varied such that $\bar{\delta}$ is always equal to0.1215 in. This represents an amplitude of approximately 2.5 times the thickness of the linear theory optimum point (t = 0.05 in.). The underlying thought here is that, in order for the comparison to be meaningful, the amplitude of imperfection must be the same for all configurations.

The geometry of all configurations employed in the comparison are given on Table 2. The symbols used are the same as those of Refs. 1,7,8 and 11. Point 1 corresponds to <u>the</u> optimum point as obtained by employing linear buckling analysis in the optimization procedure. Points 2 and 3 correspond to the lightest configuration for a given skin thickness or curvature parameter, Z. Points 1a through 1e correspond to designs surrounding the optimum point but for the same value of the curvature parameter

as the optimum point. Points 4 and 5 are discussed later.

6.2 Discussion

All of the design points, discussed above and listed on Table 2, represent the linear buckling theory optimum design (pt. 1) and designs surrounding this optimum (pts 2,3, 1a-1e). Each one of these points was then analyzed in order to find its load carrying capacity (buckling load) in the presence of the same amplitude imperfection. The linear (perfect geometry) and nonlinear (imperfect geometry) buckling loads are given on Table 2.

The goal of the entire procedure is to find the lightest possible configuration, which can carry safely a torsional load of 300 lbs/in. A comparison among all designs shows that designs 1, and 1a through 1e, all can carry safely approximately 300 lbs/in. The lightest, though, of all these designs is point 1. Therefore, one may conclude that the optimum design of the imperfect stiffened cylindrical shell is given by design point 1.

In order to strengthen the above conclusion one more question must be answered. Before posing this question, though, let us state two observations based on linear theory optimization procedures. Given a stiffened cylinder of specified radius, length and material, first by increasing the value of the applied load (small increments) the corresponding optimal weight increases; and second, the same small increments in applied load result in small variations in the optimal geometry ($\bar{\alpha}_x, \bar{\alpha}_y, \bar{\lambda}_{xx}, \bar{\lambda}_{yy}, \bar{\rho}_{xx}, \bar{\rho}_{yy}$, and Z). Having made these observations (true for other load cases as well) the question that must be answered is the following: Is it possible to decrease slightly the applied load (by increasing the knockdown factor)

and have an optimal configuration which is less imperfection sensitive, thus resulting in a lighter configuration, which can carry safely $N_{xy} = 300$ lbs/in.? Also, how imperfection sensitive is an optimal configuration obtained from the linear buckling analysis optimization with a slightly higher applied load? In order to answer these two questions, optimal configurations are generated for $\bar{N}_{xy_{cl}} = 394.7$ lbs/in. and 435.0 lbs/in., and presented as points 4 and 5 on Table 1. As expected the corresponding weights are slightly lower and higher than that of point 1, respectively. By performing a nonlinear buckling analysis on these configurations (pts. 4 and 5) it is clearly seen that both are more imperfection sensitive than that of point 1 and thus we may conclusively state that, the optimum geometry for the problem in hand is given by design point 1. Note that the last row of Table 2 gives the ratio of total weight to the critical torsion (shear resultant) for each design. This ratio can be thought of as a performance index for each design; the lower the ratio the better the design.

TABLE 2. Performance of Various Design Configurations

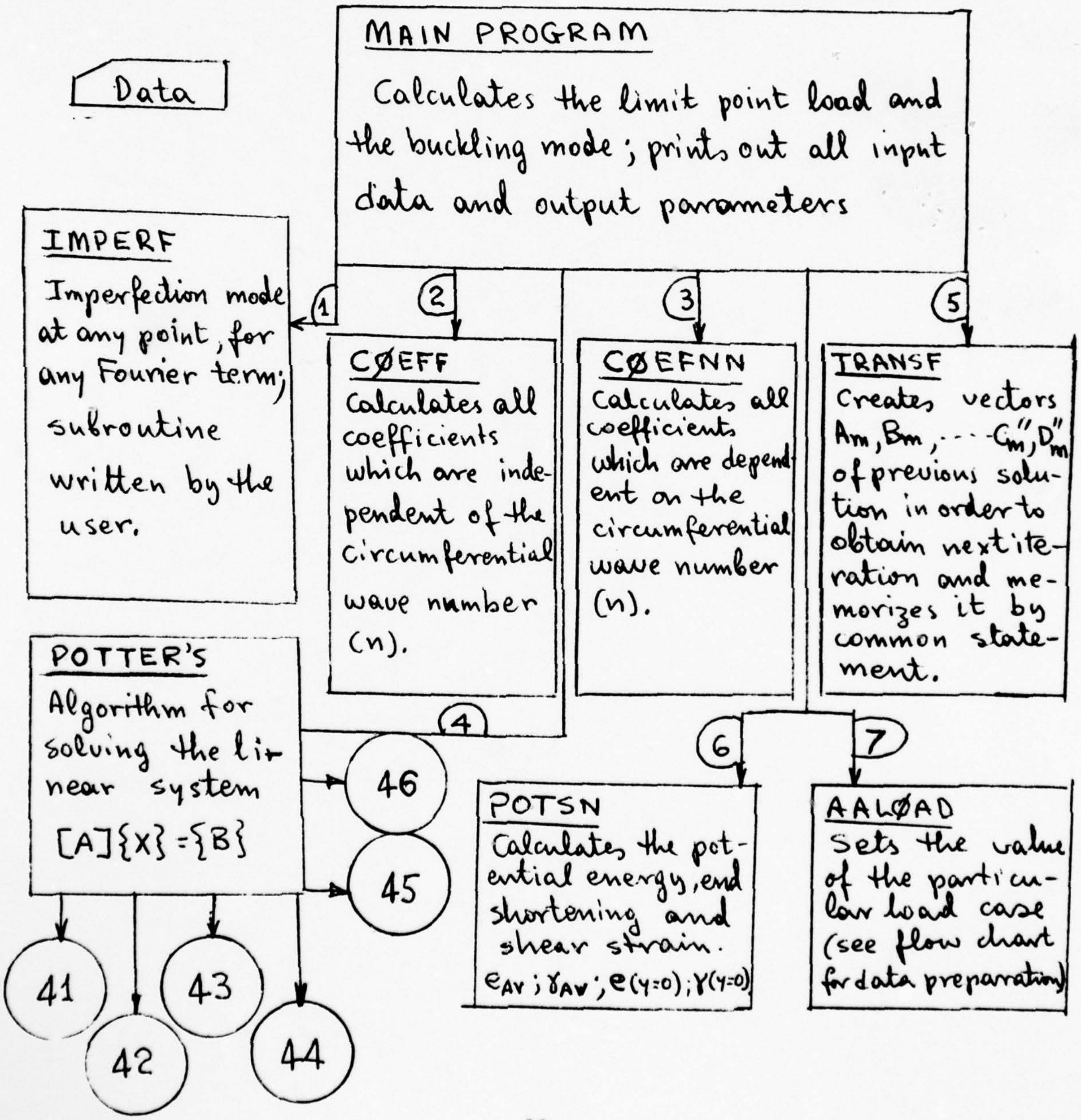
Design Pt. → / Geom. ↓	1	2	3	1a	1b	1c	1d	1e	4	5
W(lbs.)	311.2	346.2	459.9	336.5	317.2	335.51	322.6	320.4	301.25	318.37
t(in.)	0.0500	0.0555	0.0740	0.0500	0.0500	0.0500	0.0500	0.0500	0.0500	0.0500
$\bar{\alpha}_x$	7	8	7	5	6	9	10	10	5	7
$\bar{\alpha}_y$	20	17	9	21	22	18	20	22	20	21
$\bar{\lambda}_{xx}$	0.08653	0.09396	0.08831	0.17487	0.11533	0.15648	0.12574	0.12682	0.05892	0.11163
$\bar{\lambda}_{yy}$	0.05043	0.04502	0.04678	0.04562	0.04163	0.06077	0.04906	0.04054	0.04524	0.04911
$\bar{\rho}_{xx}$	4.240	6.013	4.327	4.372	4.152	12.675	12.574	12.682	1.473	5.470
$\bar{\rho}_{yy}$	20.172	13.011	3.789	20.118	20.149	19.689	19.624	19.621	18.096	21.658
e_x	0.20754	0.25943	0.30729	0.15538	0.18146	0.25969	0.28577	0.28577	0.15538	0.20754
e_y	0.52503	0.49976	0.37019	0.55003	0.57503	0.47503	0.52503	0.57503	0.52503	0.55003
Z	2221	2000	1500	2221	2221	2221	2221	2221	2221	2221
Linear $\bar{N}_{xy_{cl}}$	418.54	418.54	418.54	418.54	418.54	418.54	418.54	418.54	394.7	435
Linear n	13	13	13	13	13	13	13	12	13	13
Nonlin. $\bar{N}_{xy_{cr}}$	296	275	275	296	296	294	294	294	254	280
Nonlin. n	13	14	15	13	13	13	13	13	14	13
$\bar{N}_{xy_{cr}}/\bar{N}_{xy_{cl}}$	0.707	0.655	0.655	0.707	0.707	0.701	0.701	0.701	0.654	0.644
$W/\bar{N}_{xy_{cr}}$	1.0513	1.25890	1.67236	1.113682	1.07163	1.18971	1.14397	1.10865	1.18602	1.13603

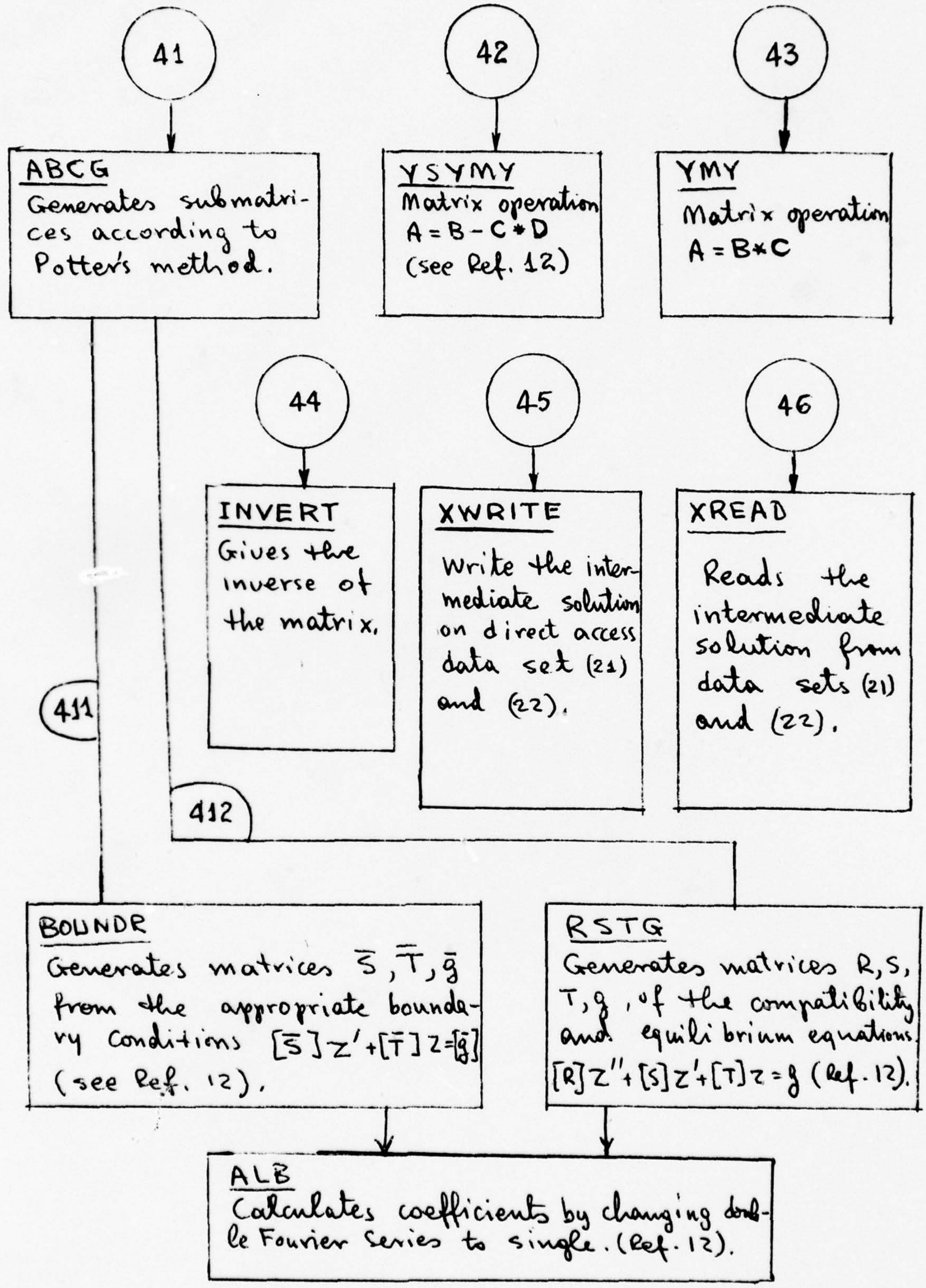
APPENDIX A

COMPUTER PROGRAM

I. BLOCK DIAGRAM.

Since this program is an extension and modification of the one employed in Ref. 12, the reader is referred to the text and Appendix B of Ref. 12.




41
42
43
ABCG
Generates submatrices according to Potter's method.
YSYMY
Matrix operation A = B - C * D (see Ref. 12)
YMY
Matrix operation A = B*C
44
45
46
INVERT
Gives the inverse of the matrix.
XWRITE
Write the intermediate solution on direct access data set (21) and (22).
XREAD
Reads the intermediate solution from data sets (21) and (22).
411
412
BOUNDR
Generates matrices $\bar{S}, \bar{T}, \bar{g}$ from the appropriate boundary conditions $[\bar{S}] Z' + [\bar{T}] Z = [\bar{g}]$ (see Ref. 12).
RSTG
Generates matrices R, S, T, g, of the compatibility and equilibrium equations $[R] Z'' + [S] Z' + [T] Z = g$ (Ref. 12).
ALB
Calculates coefficients by changing double Fourier Series to single. (Ref. 12).

COMMON CARDS

1) **CØMMØN/CINTG/NEQPØT, MI(500).**

NEQPØT – Number of points in axial direction.

MI(500) – The order, MI(I), of Eq. I of Potter's algorithm.

2) **CØMMØN/BOUND/ LS1, LSN.**

Definition of boundary conditions at the first point (LS1) and at the last point (LSN) of the shell.

3) **CØMMØN/FIDFR/DELTA, AL1, GA1, AL2, BT2, GA2**

Coefficients of finite difference form (see Ref. 12).

4) **CØMMØN/FØUR1/KFØUR, K1, K2, ... K9**

CØMMØN/FØUR2/K10, K11, ... K15, KK2.

Fourier Series limit (k = KFØUR) and parameters dependent on k.

5) **CØMMØN/GEØM/RR, DD, H11, H12, H22, Q11, Q12, Q22, D11, D12, D22.**

Shell geometric parameters $R, D, h_{ij}, q_{ij}, d_{ij}$ (Ref. 12).

6) **CØMMØN/FACTØR/C1, C2, ... C12.**

Coefficients dependent on circumferential wave number (n).

7) **CØMMØN/FACT2/DL1, ... DL4, DA1, .. DA4, DB2, .. DB4, XNI, EXXP.**

Coefficients, $\gamma_1 - \gamma_4$, $a_1 - a_4$, b_2, b_3, b_4, ν, E_{xxp} (see Ref. 12).

8) <u>CØMMØN/FACT3/XL,XH.</u>

XL - Length, XH - thickness.

9) <u>CØMMØN/CDISK/I21(501),I22(501).</u>

Direct access data sets 21 and 22.

10) <u>CØMMØN/PRES1/AWM(100,3),AWMP(100,3),AWMPP(100,3).</u>

Vectors A_m, A'_m, A''_m of previous solution

11) <u>CØMMØN/PRES2/BWM(100,2),BWMP(100,2),BWMPP(100,2).</u>

Vectors B_m, B'_m, B''_m of previous solution.

12) <u>CØMMØN/PRES3/CFM(100,4),CFMP(100,4),CFMPP(100,4).</u>

Vectors C_m, C'_m, C''_m of previous solution.

13) <u>CØMMØN/PRES4/DFM(100,4),DFMP(100,4),DFMP(100,4).</u>

vectors D_m, D'_m, D''_m of previous solution.

14) <u>CØMMØN/PRES5/AWZ(100,3),AWZP(100,3),AWZPP(100,3).</u>

Symmetric imperfection mode A^o, $A^{o\prime}$, $A^{o\prime\prime}$.

15) <u>CØMMØN/PRES6/BWZ(100,2),BWZP(100,2),BWZPP(100,2).</u>

Antisymmetric imperfection mode B^o, $B^{o\prime}$, $B^{o\prime\prime}$.

16) <u>CØMMØN/RESBN1/AWMB(2,3),AWMPPB(2,3),BWMB(2,3),BWMPPB(2,3).</u>

Vectors (A,B) of previous solution at fictitious points

17) <u>CØMMØN/RESBN2/CFMB(2,4),CFMPPB(2,4),DFMB(2,4),DFMPPB(2,4).</u>

vectors (C,D) of previous solution at fictitious points.

18) <u>CØMMØN/XXLØAD/AXPRES(100,3),BXPRES(100,2).</u>

Value of sym. and antis. pressure at pt I, in x-direction, for term j.

II. FLOW CHART FOR DATA PREPARATION.

START

TITLE (9) — 9A8

Any alphanumeric information for case title (72 columns).

NEQPØT, KFØUR, LS1, LSN, LPRINT, LMØD. — 10I6

NEQPØT – No. of points in axial direction, $5 \leq NEQPØT \leq 100$

KFØUR – No. of terms in Fourier series, $1 \leq KFØUR \leq 2$

LS1 Boundary conditions at the first point.

LSN Boundary conditions at the last point.

LS=1 Boundary Cond. SS1 ; LS=5 Boundary Cond. CC1

LS=2 " " SS2 ; LS=6 " " CC2

LS=3 " " SS3 ; LS=7 " " CC3

LS=4 " " SS4 ; LS=8 " " CC4.

LS=9 " " Free Edge.

LS=10 Symmetry Condition.

LS=11 Antisymmetry " .

} For LS = i , i = 1, 2, ... 10 see Ref. 12.

LPRINT = 1 max. printout ; = 0 min. printout

LMØD = 1 print modes ; = 0 do not print modes

A1

A1

RR, XL, XH, ELAS, XNI. — 6E12.4

RR Cylinder radius
XL Cylinder length
XH Cylinder thickness
ELAS Modulus of elasticity
XNI Poisson's ratio

XLAMD, YLAMD, EX, EY, RHØX, RHØY. — 6E12.4

$$\left.\begin{array}{l} \text{XLAMD} = \lambda_x = E_x A_x (1-\nu^2)/E t \ell_x \\ \text{YLAMD} = \lambda_y = E_y A_y (1-\nu^2)/E t \ell_y \end{array}\right\} \text{Smeared extensional stiffness of stringers and rings.}$$

$$\left.\begin{array}{l} \text{EX} \\ \text{EY} \end{array}\right\} \text{stringer and ring eccentricities (positive inward)}$$

$$\left.\begin{array}{l} \text{RHØX} = \bar{\rho}_{xx} = E_x I_{xc} / D \ell_x \\ \text{RHØY} = \bar{\rho}_{yy} = E_y J_{yc} / D \ell_y \end{array}\right\} \text{smeared flexural stiffness of stringers and rings.}$$

A2

A2

The user must write the subroutine IMPERF for definition of the imperfection and its derivatives as functions of x.

$$W^{0}(x,y) = \sum_{j=1}^{KFOUR+1} AWZ\,(I,j) \cos \frac{jny}{R} + \sum_{j=1}^{KFOUR} BWZ\,(I,j) \sin \frac{jny}{R}$$

where $AWZP = \dfrac{\partial\,(AWZ)}{\partial x}$; $AWZPP = \dfrac{\partial^2\,(AWZ)}{\partial x^2}$

$BWZP = \dfrac{\partial\,(BWZ)}{\partial x}$; $BWZPP = \dfrac{\partial^2\,(BWZ)}{\partial x^2}$

The use must write program AWZ, ... BWZPP for I=1, NEQPØT (for every point in x-direction)

j=1 , KFØUR+1 for AWZ, AWZP, AWZPP

j=1 , KFØUR for BWZ, BWZP, BWZPP.

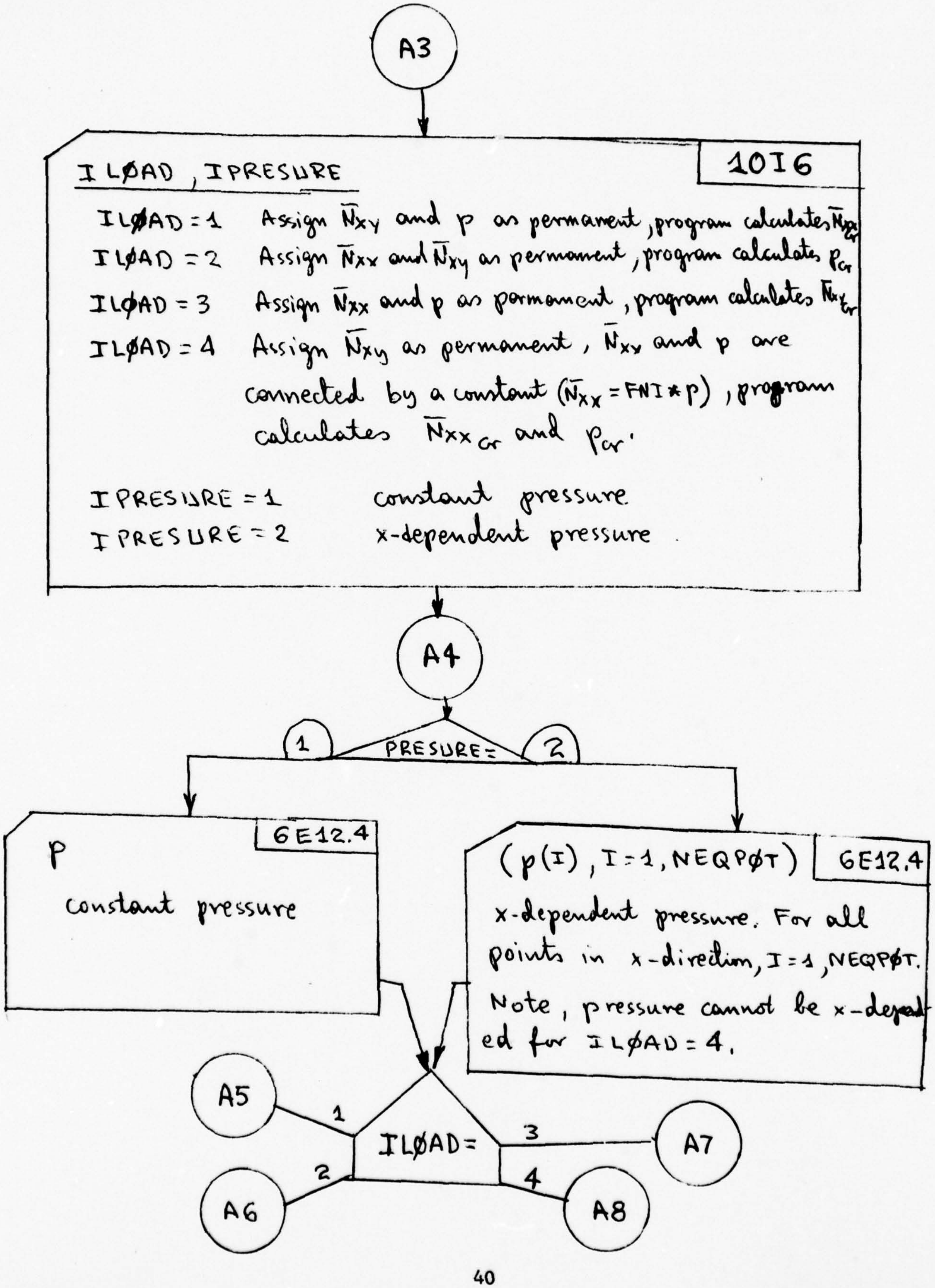
j = Fourier terms

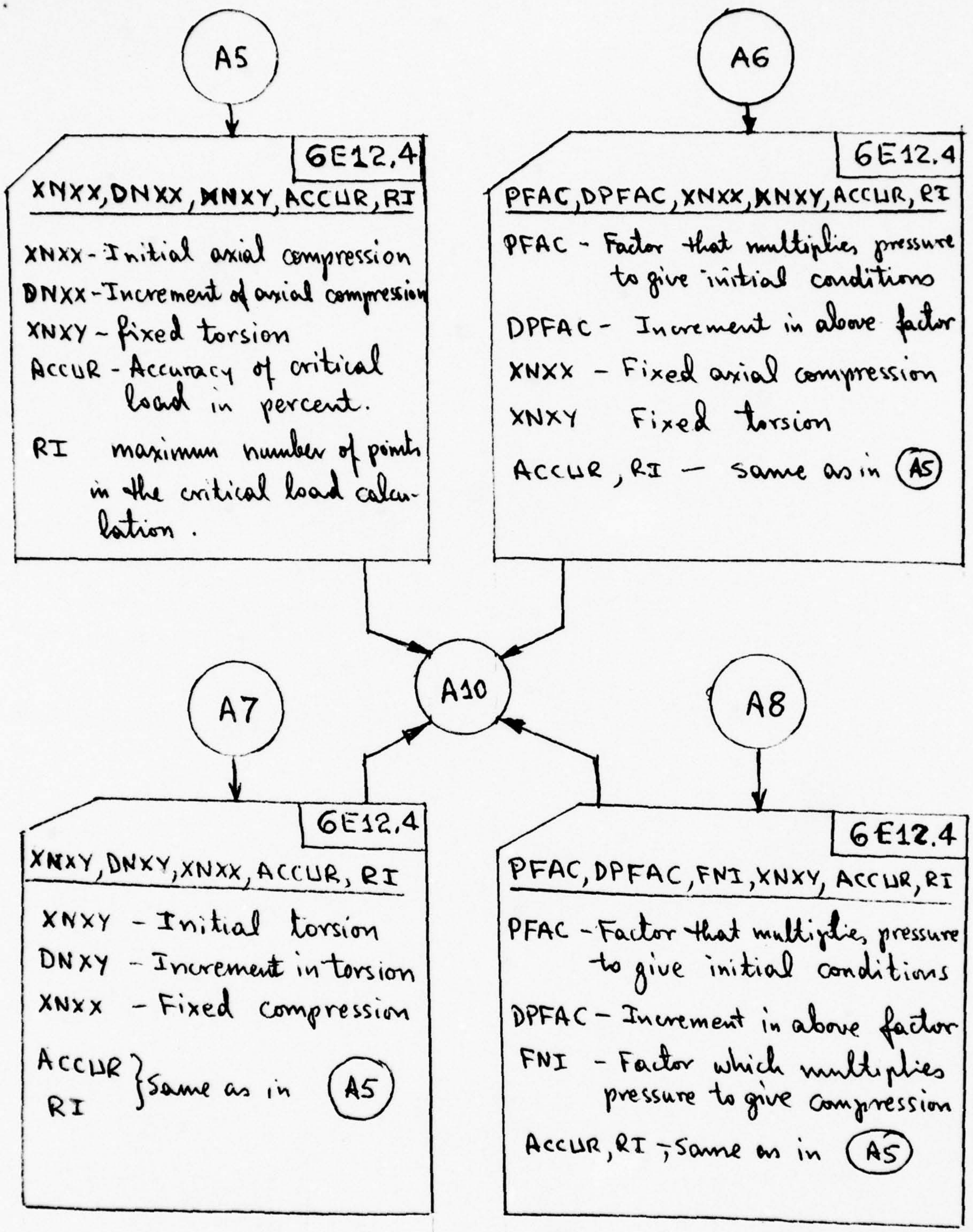
j=1 → i=0 for AWZ, AWZP, AWZPP

j=1 → i=1 for BWZ, BWZP, BWZPP.

Note: $w^{\circ}(x,y)$ is positive inward

A3


A3
I LØAD , IPRESURE
10I6
ILØAD = 1 Assign $\bar{N}_{xy}$ and p as permanent, program calculates $\bar{N}_{xx_{cr}}$
ILØAD = 2 Assign $\bar{N}_{xx}$ and $\bar{N}_{xy}$ as permanent, program calculates p_{cr}
ILØAD = 3 Assign $\bar{N}_{xx}$ and p as permanent, program calculates $\bar{N}_{xy_{cr}}$
ILØAD = 4 Assign $\bar{N}_{xy}$ as permanent, $\bar{N}_{xx}$ and p are connected by a constant ($\bar{N}_{xx}$ = FNI * p), program calculates $\bar{N}_{xx_{cr}}$ and p_{cr}.
IPRESURE = 1 constant pressure
IPRESURE = 2 x-dependent pressure
A4
PRESURE =
1
2
P
6E12.4
constant pressure
(p(I), I = 1, NEQPØT)
6E12.4
x-dependent pressure. For all points in x-direction, I = 1, NEQPØT.
Note, pressure cannot be x-depended for ILØAD = 4.
ILØAD =
1
2
3
4
A5
A6
A7
A8


A5
6E12.4
XNXX, DNXX, XNXY, ACCUR, RI
XNXX - Initial axial compression
DNXX - Increment of axial compression
XNXY - fixed torsion
ACCUR - Accuracy of critical load in percent.
RI maximum number of points in the critical load calculation.
A6
6E12.4
PFAC, DPFAC, XNXX, XNXY, ACCUR, RI
PFAC - Factor that multiplies pressure to give initial conditions
DPFAC - Increment in above factor
XNXX - Fixed axial compression
XNXY Fixed torsion
ACCUR, RI — same as in A5
A10
A7
6E12.4
XNXY, DNXY, XNXX, ACCUR, RI
XNXY - Initial torsion
DNXY - Increment in torsion
XNXX - Fixed compression
ACCUR RI Same as in A5
A8
6E12.4
PFAC, DPFAC, FNI, XNXY, ACCUR, RI
PFAC - Factor that multiplies pressure to give initial conditions
DPFAC - Increment in above factor
FNI - Factor which multiplies pressure to give compression
ACCUR, RI - Same as in A5

NNN, LNNN, ILNW

10I6

NNN (n) The program finds the limit point for this n.

LNNN = 0 The program does not check which n-values gives minimum potential (total)

LNNN = 1 The program finds the limit point for NNN, calculates the total potential for n-values surrounding NNN and load levels lower than the limit point of NNN.

LNNN = 2 The program calculates the total potential only at the initial (for the step) load level.

ILNW ≤ 10 The upper limit on n for which the program calculates the total potential in order to find which n gives minimum total potential.

LCØNTN

10I6

LCØNTN = 1 There is another example to run.

LCØNTN ≠ 1 This is the last example.

III COMPUTER PROGRAM

```
      PROGRAM MAIN(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT,TAPE21,TAPE22)
C     MAIN
C   POST BUCKLING OF STIFFENED CYLINDRICAL SHELLS UNDER COMBINED LOAD OF
C   UNIFORM AXIAL LOAD, LATERAL LOAD AND TORSION
      COMMON/FOUR1/KFOUR,K1,K2,K3,K4,K5,K6,K7,K8,K9
      COMMON/FOUR2/K10,K11,K12,K13,K14,K15,KK2
      COMMON/CINTG/NEQPOT,MI(500)
      COMMON/CDISK/I21(501),I22(501)
      COMMON/FIDFR/DELTA,AL1,GA1,AL2,BT2,GA2
      COMMON/PRES1/AWM(100,3),AWMP(100,3),AWMPP(100,3)
      COMMON/PRES2/BWM(100,2),BWMP(100,2),BWMPP(100,2)
      COMMON/PRES3/CFM(100,4),CFMP(100,4),CFMPP(100,4)
      COMMON/PRES4/DFM(100,4),DFMP(100,4),DFMPP(100,4)
      COMMON/PRES5/AWZ(100,3),AWZP(100,3),AWZPP(100,3)
      COMMON/PRES6/BWZ(100,2),BWZP(100,2),BWZPP(100,2)
      COMMON/RESBN1/AWMB(2,3),AWMPPB(2,3),BWMB(2,2),BWMPPB(2,2)
      COMMON/RESBN2/CFMB(2,4),CFMPPB(2,4),DFMB(2,4),DFMPPB(2,4)
      COMMON/GEOM/RR,DD,H11,H12,H22,Q11,Q12,Q22,D11,D12,D22
      COMMON/FACTOR/C1,C2,C3,C4,C5,C6,C7,C8,C9,C10,C11,C12
      COMMON/FACT2/DL1,DL2,DL3,DL4,DA1,DA2,DA3,DA4,DB2,DB3,DB4,XNI,EXXP
      COMMON/FACT3/XL,XH
      COMMON/XXLOAD/AXPRES(100,3),BXPRES(100,2)
      COMMON/BOUND/LS1,LSN
      DIMENSION TI(10),XXPRES(100,3)
      DIMENSION AP(52,52),BP(52,52),CP(52,52),PR(52,52),GP(52,1)
      DIMENSION XP(52,1),T1(52),CC(52),MT(52),V1(2704)
C   2704=52*52
      DIMENSION AWWM(3),BWWM(2),CCM(4),DDM(4)
      DIMENSION AWCON(20,3),BWCON(20,2),CCON(20,4),DDON(20,4)
      EQUIVALENCE (AP(1,1),V1(1))
      CALL OPENMS(21,I21,501,0)
      CALL OPENMS(22,I22,501,0)
      ECONV=0.000001
      MAXN=52
      MAX2=MAXN*MAXN
      NRHS=1
      NJ=100
      NAW=3
      NBW=2
      NF=4
      M3=100
      M4=3
 1111 WRITE(6,20)
      READ(5,10)(T1(I),I=1,9)
      WRITE(6,60)
      WRITE(6,10)(TI(I),I=1,9)
      READ(5,100)NEQPOT,KFOUR,LS1,LSN,LPRINT,LMOD
      IF(LPRINT.EQ.1)LMOD=1
      IDDET=1
      READ(5,200)RR,XL,XH,ELAS,XNI
      READ(5,200)XLAMD,YLAMD,EX,EY,RHOX,RHOY
C   ********************************************************************
      CALL COEFF(EX,EY,XLAMD,YLAMD,RHOX,RHOY,ELAS)
```

```
C  ********************************************************************************
      WRITE(6,300)NEQPOT,KFOUR,LS1,LSN
      WRITE(6,400)RR,XL,XH,ELAS,XNI,DO,EXXP
      WRITE(6,573)XLAMD,YLAMD,EX,EY,RHOX,RHOY
C  ********************************************************************************
      CALL IMPERF
C  ********************************************************************************
      WRITE(6,508)
      J1=0
      WRITE(6,510)J1
      WRITE(6,520)
      XX=0.
      DO 85 I1=1,NEQPOT
      WRITE(6,509)I1,XX,AWZ(I1,1),AWZP(I1,1),AWZPP(I1,1)
      XX=XX+DELTA
   85 CONTINUE
      DO 86 J1=1,KFOUR
      WRITE(6,510)J1
      WRITE(6,520)
      XX=0.
      DO 86 I1=1,NEQPOT
      WRITE(6,5091)I1,XX,AWZ(I1,J1+1),AWZP(I1,J1+1),AWZPP(I1,J1+1),
     1BWZ(I1,J1),BWZP(I1,J1),BWZPP(I1,J1)
      XX=XX+DELTA
   86 CONTINUE
      IF(LPRINT.NE.1)GO TO 39
      WRITE(6,500)DELTA,AL1,GA1,AL2,BT2,GA2
      WRITE(6,501)H11,H12,H22,Q11,Q12,Q22
      WRITE(6,502)D11,D12,D22,DB2,DB3,DB4
      WRITE(6,503)DL1,DL2,DL3,DL4
      WRITE(6,504)DA1,DA2,DA3,DA4
   39 CONTINUE
      DO 63 I1=1,NEQPOT
      DO 64 J1=1,K1
      AWM(I1,J1)=0.
      AWMP(I1,J1)=0.
      AWMPP(I1,J1)=0.
      AXPRES(I1,J1)=0.
      XXPRES(I1,J1)=0.
      IF(J1.EQ.K1)GO TO 64
      BWM(I1,J1)=0.
      BWMP(I1,J1)=0.
      BWMPP(I1,J1)=0.
      BXPRES(I1,J1)=0.
   64 CONTINUE
      DO 65 J1=1,KK2
      CFM(I1,J1)=0.
      CFMP(I1,J1)=0.
      CFMPP(I1,J1)=0.
      DFM(I1,J1)=0.
      DFMP(I1,J1)=0.
      DFMPP(I1,J1)=0.
   65 CONTINUE
```

```
   63 CONTINUE
      READ(5,100)ILOAD,IPRESS
      IF(IPRESS.EQ.2)GO TO 637
      READ(5,200)P11
      DO 639 I1=1,NEQPOT
  639 XXPRES(I1,1)=P11
      GO TO 640
  637 READ(5,200)(XXPRES(I1,1),I1=1,NEQPOT)
  640 CONTINUE
      WRITE(6,2001)
      WRITE(6,2002)(I1,XXPRES(I1,1),I1=1,NEQPOT)
      GO TO (1,2,3,4),ILOAD
    1 READ(5,200)XNXXA,DNXXA,XNXYA,ACCUR,RII
      WRITE(6,511)XNXXA,DNXXA,XNXYA,ACCUR
      DO 11 I1=1,NEQPOT
   11 AXPRES(I1,1)=XXPRES(I1,1)
      XNX=XNXXA
      DNX=DNXXA
      PFAC=1.
      XN11=XNXXA
      GO TO 5
    2 READ(5,200)PFAC,DPFAC,XNXXA,XNXYA,ACCUR,RII
      WRITE(6,512)PFAC,DPFAC,XNXXA,XNXYA,ACCUR
      DO 12 I1=1,NEQPOT
   12 AXPRES(I1,1)=PFAC*XXPRES(I1,1)
      XNX=PFAC
      DNX=DPFAC
      XN11=PFAC
      GO TO 5
    3 READ(5,200)XNXYA,DNXYA,XNXXA,ACCUR,RII
      WRITE(6,513)XNXYA,DNXYA,XNXXA,ACCUR
      DO 13 I1=1,NEQPOT
   13 AXPRES(I1,1)=XXPRES(I1,1)
      XNX=XNXYA
      DNX=DNXYA
      PFAC=1.
      XN11=XNXYA
      GO TO 5
    4 READ(5,200)PFAC,DPFAC,FNI,XNXYA,ACCUR,RII
      DO 14 I1=1,NEQPOT
   14 AXPRES(I1,1)=PFAC*XXPRES(I1,1)
      XNXXA=FNI*AXPRES(1,1)
      DNXXA=DPFAC*XNXXA
      XNX=PFAC
      DNX=DPFAC
      P1=AXPRES(1,1)
      DP1=DPFAC*P1
      XN11=PFAC
      WRITE(6,514)P1,DP1,FNI,XNXXA,XNXYA,ACCUR
    5 IRR=RII
      IF(IRR.EQ.0)IRR=1
      XFNX=XNXXA
      XFNY=XNXYA
```

```
      XFNP=PFAC
      TXNX=1000.*XNX
      READ(5,100)NNN,LNNN,ILNW
      NWAVE=NNN
C     ************************************************************************
      CALL COEFNN(NWAVE)
C     ************************************************************************
      WRITE(6,505)NNN
      IF(LPRINT.NE.1)GO TO 49
      WRITE(6,506)C1,C2,C3,C4,C5,C6
      WRITE(6,507)C7,C8,C9,C10,C11,C12
   49 CONTINUE
      ILR=0
      LICON=1
      IPOTT=0
      CALL SECOND(TIM1)
      WRITE(6,793)TIM1
      TIM2=TIM1
      TIM4=TIM2
      IINN=0
  555 LN=1
      CALL AALOAD(ILOAD,NEQPOT,FNI,XNX,XFNX,XFNY,XFNP,XXPRES,AXPRES,M3,
     1M4)
      IDET=IDDET
      CALL POTERS(IDET,NRHS,MAXN,AP,BP,CP,GP,PR,XP,CC,MT,T1,V1,MAX2,
     1IXPM,DETM,XFNX,XFNY,LN,NJ,NAW,NBW,NF)
      IF(LPRINT.NE.1)GO TO 101
      CALL SECOND(TIM3)
      TIM1=TIM3-TIM2
      TIM2=TIM3
      TIM4=TIM3
      WRITE(6,201)NWAVE,XFNX,XFNY,XFNP,TIM1
  101 CALL TRANSF(T1,MAXN,1,LPRINT)
      IDET=IDDET
      CALL AALOAD(ILOAD,NEQPOT,FNI,XNX,XFNX,XFNY,XFNP,XXPRES,AXPRES,M3,
     1M4)
      IAMAX=1
      IBMAX=1
      AWMAX=0.
      BWMAX=0.
      ITER=0
      DO 102 J1=1,K1
      AWMAX=AWMAX+AWM(1,J1)
      IF(J1.EQ.K1) GO TO 102
      BWMAX=BWMAX+BWM(1,J1)
  102 CONTINUE
         DO 103 I1=2,NEQPOT
      AWMM=0.
      BWMM=0.
      DO 104 J1=1,K1
      AWMM=AWMM+AWM(I1,J1)
      IF(J1.EQ.K1) GO TO 104
      BWMM=BWMM+BWM(I1,J1)
```

```
 104 CONTINUE
     IF(ABS(AWMM).LE.ABS(AWMAX)) GO TO 32
     AWMAX=AWMM
     IAMAX=I1
  32 IF(ABS(BWMM).LE.ABS(BWMAX)) GO TO 103
     BWMAX=BWMM
     IBMAX=I1
 103 CONTINUE
     JAWMAX=1
     JBWMAX=1
     AWWM(1)=AWM(IAMAX,1)
     AAWWM=AWWM(1)
     BWWM(1)=BWM(IBMAX,1)
     BAWWM=BWWM(1)
     IF(K1.EQ.1)GO TO 1051
     DO 105 J1=2,K1
     AWWM(J1)=AWM(IAMAX,J1)
     IF(ABS(AWWM(J1)).LE.ABS(AAWWM))GO TO 31
     AAWWM=AWWM(J1)
     JAWMAX=J1
  31 IF(J1.EQ.K1)GO TO 105
     BWWM(J1)=BWM(IBMAX,J1)
     IF(ABS(BWWM(J1)).LE.ABS(BAWWM))GO TO 105
     BAWWM=BWWM(J1)
     JBWMAX=J1
 105 CONTINUE
1051 JCMAX=1
     JDMAX=1
     CCM(1)=CFM(IAMAX,1)
     DDM(1)=DFM(IBMAX,1)
     ACCM=CCM(1)
     ADDM=DDM(1)
     IF(KK2.EQ.1)GO TO 333
     DO 106 J1=2,KK2
     CCM(J1)=CFM(IAMAX,J1)
     DDM(J1)=DFM(IBMAX,J1)
     IF(ABS(CCM(J1)).LE.ABS(ACCM))GO TO 30
     ACCM=CCM(J1)
     JCMAX=J1
  30 IF(ABS(DDM(J1)).LE.ABS(ADDM))GO TO 106
     ADDM=DDM(J1)
     JDMAX=J1
 106 CONTINUE
 333 LN=2
     ITER=ITER+1
     IF(ITER.LE.10)GO TO 113
     WRITE(6,114)ITER
     GO TO 9999
 113 CALL POTERS(IDET,NRHS,MAXN,AP,BP,CP,GP,PR,XP,CC,MT,T1,V1,MAX2,
    1IXPM,DETM,XFNX,XFNY,LN,NJ,NAW,NBW,NF)
     IF(LPRINT.NE.1)GO TO 111
     CALL SECOND(TIM3)
     TIM1=TIM3-TIM2
```

```
      TIM2=TIM3
      WRITE(6,112)ITER,NWAVE,XFNX,XFNY,XFNP,TIM1
  111 CALL TRANSF(T1,MAXN,1,LPRINT)
      DO 115 J1=1,K1
      IF(AWM(IAMAX,J1).NE.0)GO TO 58
      AWCON(ITER,J1)=0.
      GO TO 56
   58 AWCON(ITER,J1)=ABS((AWM(IAMAX,J1)-AWWM(J1))/AWM(IAMAX,J1))
   56 IF(J1.EQ.K1)GO TO 115
      IF(BWM(IBMAX,J1).NE.0.)GO TO 57
      BWCON(ITER,J1)=0.
      GO TO 115
   57 CONTINUE
      BWCON(ITER,J1)=ABS((BWM(IBMAX,J1)-BWWM(J1))/BWM(IBMAX,J1))
  115 CONTINUE
      AWCH=AWCON(ITER,JAWMAX)
      BWCH=BWCON(ITER,JBWMAX)
      IAWH=JAWMAX
      IBWH=JBWMAX
      DO 116 J1=1,KK2
      IF(CFM(IAMAX,J1).NE.0.)GO TO 54
      CCON(ITER,J1)=0.
      GO TO 53
   54 CCON(ITER,J1)=ABS((CFM(IAMAX,J1)-CCM(J1))/CFM(IAMAX,J1))
   53 IF(DFM(IBMAX,J1).NE.0.)GO TO 52
      DDON(ITER,J1)=0.
      GO TO 116
   52 DDON(ITER,J1)=ABS((DFM(IBMAX,J1)-DDM(J1))/DFM(IBMAX,J1))
  116 CONTINUE
      CCH=CCON(ITER,JCMAX)
      DDH=DDON(ITER,JDMAX)
      ICH=JCMAX
      IDH=JDMAX
      IF(LPRINT.NE.1)GO TO 117
      WRITE(6,118)ITER,AWCH,BWCH,CCH,DDH
      WRITE(6,119)(J1,AWCON(ITER,J1),J1=1,K1)
      WRITE(6,119)(J1,BWCON(ITER,J1),J1=1,KFOUR)
      WRITE(6,119)(J1,CCON(ITER,J1),J1=1,KK2)
      WRITE(6,119)(J1,DDON(ITER,J1),J1=1,KK2)
  117 IF(AWCH.GT.ECONV)GO TO 194
      IF(BWCH.GT.ECONV)GO TO 194
      IF(CCH.GT.ECONV)GO TO 194
      IF(DDH.GT.ECONV)GO TO 194
      IF(XNX.EQ.XN11) GO TO 193
      IF(DPMS/DETM.GT.0.) GO TO 193
      WRITE(6,192)DETM,IXPM
      GO TO 197
  193 DPMS=DETM
      GO TO 195
  194 IF(ITER.LE.2)GO TO 196
      IF(AWCON(ITER,IAWH).GT.AWCON(ITER-1,IAWH))GO TO 197
      IF(BWCON(ITER,IBWH).GT.BWCON(ITER-1,IBWH))GO TO 197
      IF(CCON(ITER,ICH).GT.CCON(ITER-1,ICH))GO TO 197
```

```
      IF(DDON(ITER,IDH).GT.DDON(ITER-1,IDH))GO TO 197
      GO TO 196
197   IF(XNX.NE.XN11)GO TO 198
      WRITE(6,991)XNX
      GO TO 9999
196   DO 131 J1=1,K1
      AWWM(J1)=AWM(IAMAX,J1)
      IF(J1.EQ.K1)GO TO 131
      BWWM(J1)=BWM(IBMAX,J1)
131   CONTINUE
      DO 132 J1=1,KK2
      CCM(J1)=CFM(IAMAX,J1)
132   DDM(J1)=DFM(IBMAX,J1)
      GO TO 333
195   CALL AALOAD(ILOAD,NEQPOT,FNI,XNX,XFNX,XFNY,XFNP,XXPRES,AXPRES,M3,
     1M4)
      CALL POTSN(POT,STRYU,STRAU,STRYG,STRAG,XFNX,XFNY)
      CALL SECOND(TIM3)
      TIM1=TIM3-TIM4
      TIM2=TIM3
      TIM4=TIM3
      WRITE(6,241)XFNX,XFNY,XFNP,NWAVE,ITER,TIM1
      WRITE(6,242)POT,STRYU,STRAU,STRYG,STRAG
      IF(IDET.EQ.1)WRITE(6,243)DETM,IXPM
      IF(LMOD.NE.1)GO TO 476
      CALL TRANSF(T1,MAXN,2,3)
476   CONTINUE
      IF(LNNN.EQ.2.AND.LICON.NE.10)GO TO 566
      IF(LICON.NE.10)GO TO 629
      ILR=1
      GO TO 777
629   XNX1=XNX
      IINN=IINN+1
      IF(IINN.LE.IRR)GO TO 721
      WRITE(6,722)IINN
      GO TO 9999
721   CONTINUE
      XNX=XNX+DNX
      IF(TXNX.GT.XNX)GO TO 244
      DNX=DNX/2.
      XNX=XNX-DNX
      ADN=DNX*100/XNX
      IINN=IINN-1
      IF(ADN.GT.ACCUR)GO TO 244
      XNX=XNX1
      GO TO 819
244   GO TO 555
198   IF(LICON.NE.10)GO TO 429
      ILR=0
      GO TO 777
429   IF(LNNN.EQ.0)GO TO 249
      IPOTT=IPOTT+1
      IF(IPOTT.GT.1)GO TO 249
```

```
      POTT=POT
      PXNX=XNX-DNX
  249 ADN=DNX*100./XNX
      WRITE(6,545)NWAVE,XNX
      TXNX=XNX
      XNX=XNX-DNX
      IINN=0
      IF(ADN.LE.ACCUR)GO TO 819
      DNX=DNX/2.
      XNX=XNX+DNX
      GO TO 555
 819  CALL AALOAD(ILOAD,NEQPOT,FNI,XNX,XFNX,XFNY,XFNP,XXPRES,AXPRES,M3,
     1M4)
      WRITE(6,785)NWAVE,XFNX,XFNY,XFNP,POT,STRYU,STRAU,STRYG,STRAG
C  CALCULATION OF CRITICAL WAVE NUMBER
  566 WRITE(6,20)
      IF(LNNN.EQ.0)GO TO 9999
      WRITE(6,584)
      ILR=1
      NWPRIN=NWAVE
      NWAVE=NWAVE+1
      IF(LNNN.EQ.2)POTT=POT
      POTMIN=POTT
      NMIN=NNN
      INWAVE=0
      ISTOP=0
      I9=0
      POT=POTT
  777 CALL AALOAD(ILOAD,NEQPOT,FNI,XNX,XFNX,XFNY,XFNP,XXPRES,AXPRES,
     1M3,M4)
      IF(ILR.EQ.1)GO TO 778
      WRITE(6,582)NWAVE,XFNX,XFNY,XFNP
      GO TO 9999
  778 INWAVE=INWAVE+1
      IINN=0
      IF(LNNN.EQ.2)PXNX=XNX
      XNX=PXNX
      LICON=10
      IF(INWAVE.EQ.1)GO TO 391
      NWPRIN=NWAVE
      IF(I9.GE.1)GO TO 694
      IF(POTMIN.LE.POT)GO TO 139
      POTMIN=POT
      NMIN=NWAVE
      NWAVE=NWAVE+1
      GO TO 391
  139 IF(NWAVE.LE.NNN+1)GO TO 549
      ISTOP=1
      GO TO 135
  549 I9=I9+1
      IF(I9.GT.1)GO TO 694
      NWAVE=NNN-1
      GO TO 391
```

```
694 IF(POTMIN.LE.POT)GO TO 695
    POTMIN=POT
    NMIN=NWAVE
    NWAVE=NWAVE-1
    GO TO 391
695 ISTOP=1
    GO TO 185
391 CALL COEFNN(NWAVE)
    CALL SECOND(TIM2)
185 WRITE(6,581)NWPRIN,PXNX,POT,TIM2
    IF(NWAVE.LE.0)GO TO 9999
    IF(ISTOP.NE.1)GO TO 798
    WRITE(6,789)XNX,POTMIN,NMIN
    GO TO 9999
798 IF(INWAVE.GT.ILNW)GO TO 9999
    GO TO 555
9999 READ(5,100)LCONTN
    IF(LCONTN.EQ.1)GO TO 1111
 20 FORMAT(1H1)
 10 FORMAT(1H0,9A8)
 60 FORMAT(//25H BEGINNING OF NEXT CASE      ///)
100 FORMAT(10I6)
200 FORMAT(6E12.4)
300 FORMAT(//,2X,"NO. OF POINTS=",I8,2X,"KFOUR=",I8,2X,"BOUND.CON OF
   1 POINT 1=",I8,2X,"BOUND.CON. OF POINT NEQPOT=",I8)
400 FORMAT(//,2X,"R=",E12.4,2X,"XL=",E12.4,2X,"XH=",E12.4, 2X,"ELAS=",
   2E12.4,2X,"XNI=",E12.4,2X,"DO=",E12.4,2X,"EXXP=",E12.4)
573 FORMAT(//,2X,"XLAMD=",E12.4,2X,"YLAMD=",E12.4,2X,"EX=",E12.4,2X,
   1"EY=",E12.4,2X,"RHOX=",E12.4,2X,"RHOY=",E12.4)
508 FORMAT(//,2X,"THE IMPERFECTION SHAPE IN AXIAL DIRECTION IS"/)
510 FORMAT(//,2X,"THE IMPERFECTION FOR CIRCUMFERENTIAL WAVE ",I6/)
520 FORMAT(//,4X,"POINT",9X,"LENGTH",10X,"WZCOS",12X,"WZPCOS",10X,
   1"WZPPCOS",10X,"WZSIN",10X,"WZPSIN",10X,"WZPPSIN"/)
509 FORMAT(I10,4E16.6)
5091 FORMAT(I10,7E16.6)
500 FORMAT(//,2X,"DELTA=",E12.4,2X,"AL1=",E12.4,2X,"GA1=",E12.4,2X,
   2"AL2=",E12.4,2X,"BT2=",E12.4,2X,"GA2=",E12.4)
501 FORMAT(//,2X,"H11=",E12.4,2X,"H12=",E12.4,2X,"H22=",E12.4,2X,
   1"Q11=",E12.4,2X,"Q12=",E12.4,2X,"Q22=",E12.4)
502 FORMAT(//,2X,"D11=",E12.4,2X,"D12=",E12.4,2X,"D22=",E12.4,2X,"DB2
   1=",E12.4,2X,"DB3=",E12.4,2X,"DB4=",E12.4)
503 FORMAT(//,2X,"DL1=",E12.4,2X,"DL2=",E12.4,2X,"DL3=",E12.4,2X,
   1"DL4=",E12.4)
504 FORMAT(//,2X,"DA1=",E12.4,2X,"DA2=",E12.4,2X,"DA3=",E12.4,2X,"DA4=
   1",E12.4)
112 FORMAT(//,2X,"SOLUTION FOR ITERATION",I8,2X,"N=",I8,2X,
   1"NXX=",E12.4,2X,"NXY=",E12.4,"FAC.LAT.LOA=",E12.4/2X,
   2"TIME COMPUTATION =",E12.4,2X,"SECONDS"/2X,"======================
   3==================================================================
   4==========================")
118 FORMAT(//,2X,"ITER=",I8,2X,"AWCH=",E15.5,2X,"BWCH=",E15.5,
   12X,"CCH=",E15.5,2X,"DDH=",E15.5/)
119 FORMAT(4(I8,E15.5))
```

```
991 FORMAT(//,2X,"THE SOLUTION IS NOT CONVERGED, APPARENTLY THE
   1INITIAL LOAD SHOULD BE LESS THAN",E12.4)
241 FORMAT(//,2X,"NONLINEAR SOLUTION FOR NXX=",E12.4,2X,"NXY=",E12.4,
   12X,"FAC.LAT.LOA=",E12.4,2X,"WAVE=",I8,2X,"IS OBTAINED BY",I8,
   22X,"ITERATIONS"/2X,"TIME COMPUTATION=",E12.4,2X,"SECONDS"/2X,"
   3OKOKOKOKOKOKOKOKOKOKOKOKOKOKOKOKOKOKOKOKOKOKOKOKOKOKOKOKOKOKOKOKOK
   4OKOKOKOKOKOKOKOKOKOKOKOKOKOKOKOKOKOKOKOKOKOKOKOKOKOKOKOKOK")
192 FORMAT(//,2X,"THE DETERMINANT CHANGES THE SIGN  DETM=",E12.4,
   12X,"IXPM=",I10)
242 FORMAT(//,2X,"POTENTIAL ENERGY=",E17.7,2X,"END SHORTENING U FOR
   1Y=0. IS",E15.5,2X,"AVERAGE END SHOR. U=",E15.5/2X,"END SHORT.
   2 GAMA FOR Y=0. IS",E15.5,2X,"AVERAGE END SHORT. GAMA=",E15.5)
722 FORMAT(//,2X,"END OF THIS CASE BECAUSE NUMBER OF LOAD POINTS
   1 IS GREATER THAN",I8)
545 FORMAT(///,2X,"FOR WAVE NUMBER",I8,2X,"THE LOAD", E12.4,2X,"IS
   1 OVER THE LIMIT POINT")
785 FORMAT(//,2X,"CRITICAL LOAD FOR WAVE=",I8,2X,"ARE NXX=",E12.4,2X,
   1"NXY=",E12.4,2X,"LAT.LOAD=",E12.4/2X,"POTENTIAL ENERGY=",E17.7,2X,
   2"END.SHOR. U FOR Y=0. IS",E15.5,2X,"AVERAGE END. SHOR. U=",E15.5/
   32X,"END.SHOR. GAMA FOR Y=0. IS",E15.5,2X,"AVERAGE.END.SHOR.GAMA=",
   4E15.5/2X,"OKOKOKOKOKOKOKOKOKOKOKOKOKOKOKOKOKOKOKOKOKOKOKOKOKOKOKOK
   5OKOKOKOK")
584 FORMAT(//,2X,"DETERMINATION OF THE CRITICAL CERCUMFERENTIAL WAVE
   1 NUMBER (N)"///2X,"**********************************************
   2*************************"/2X,"**************************************
   3**********************************")
582 FORMAT(//,2X,"FOR WAVE=",I8,2X,"NXX=",E12.4,2X,"NXY=",E12.4,2X,
   1"LAT.LOAD=",E12.4,2X,"THE SOLUTION IS NOT CONVERGED"/2X,"
   2PROBABLY THE CRITICAL LOAD IS SMALLER THAN THESE LOADS")
581 FORMAT(//,2X,"WAVE NO.=",I10,2X,"LOAD=",E15.5,2X,"POTENTIAL ENERGY
   1=",E17.7,2X,"ELAPSED TIME=",E12.4,2X,"SECONDS")
789 FORMAT(//,2X,"FOR LOAD",E12.4,2X,"THE MINIMUM POTENTIAL ENERGY
   1 IS",E17.7,2X,"AND THE CRITICAL WAVE NUMBER IS ",I8)
511 FORMAT(//,2X,"THE INITIAL AXIAL COMPRESSION IS NXX=",E12.4,2X,
   1"THE INCREMENT IS DNXX=",E12.4/2X,"THE LATERAL LOAD IS FIXED AND
   2 THE PERMANENT TORSION IS",E12.4/2X,"THE ACCURACY=",E12.4,2X,
   3"PERCENTS"/)
512 FORMAT(//,2X,"THE FACTOR OF INITIAL LATERAL LOAD IS",E12.4,2X,"
   1THE FACTOR OF INCREMENT IS",E12.4/2X,"THE FIXED AXIAL COMPRESSION
   2IS",E12.4,"AND THE FIXED TORSION IS",E12.4/2X,"THE ACCURACY=",
   3E12.4,2X,"PERCENT"/)
513 FORMAT(//,2X,"THE INITIAL TORSION =",E12.4/2X,"THE TORSION INCREME
   1NT =",E12.4/2X,"THE FIXED AXIAL LOAD=",E12.4/2X,"THE ACCURACY =",
   2E12.4,2X,"PERCENTS"/)
514 FORMAT(//,2X,"THE LATERAL LOAD IS",E12.4,2X,"THE INCREMENT IS",E
   112.4/2X,"THE AXIAL COMPRESSION AND THE LATERAL LOAD ARE
   2 CONNECTED BY THE FACTOR",E12.4/2X,"THE INITIAL AXIAL COMPRESSION
   3 IS",E12.4,2X,"THE FIXED TORSION IS",E12.4/2X,"THE ACCURACY IS",
   4E12.4,2X,"PERCENT"/)
505 FORMAT(//,2X,"THE CIRCUMFERENTIAL WAVE NUMBER=",I6)
506 FORMAT(//,2X,"C1=",E12.4,2X,"C2=",E12.4,2X,"C3=",E12.4,2X,"C4=",
   1E12.4,2X,"C5=",E12.4,2X,"C5 6=",E12.4)
507 FORMAT(//,2X,"C7=",E12.4,2X,"C8=",E12.4,2X,"C9=",E12.4,2X,
```

```
     1"C10=",E12.4,2X,"C11=",E12.4,2X,"C12=",E12.4)
 793 FORMAT(//,2X,"ELAPSED TIME=",E12.4,2X,"SECONDS"/)
 114 FORMAT(//,2X,"END OF THIS CASE BECAUSE ITER GREATER THAN ",I8)
 243 FORMAT(//,2X,"DETERMINANT=",E12.4,2X,"IXPM=",I10)
 201 FORMAT(//,2X,"INITIAL SOLUTION (FROM LINEAR) FOR N=",I8,2X,
     1"NXX=",E12.4,2X,"NXY=",E12.4,2X,"LAT.LOAD=",E12.4/2X,"TIME
     2 COMPUTATION=",E12.4,2X,"SECONDS"/2X,"=============================
     3======================================================================
     4=======================================")
2001 FORMAT(//,2X,"POINT",9X,"PRESSURE"/)
2002 FORMAT(I10,E12.4)
     END
```

```
      SUBROUTINE AALOAD(ILOAD,NEQPOT,FNI,XNX,XFNX,XFNY,XFNP,
     1XXPRES,AXPRES,M1,M2)
      DIMENSION XXPRES(M1,M2),AXPRES(M1,M2)
      GO TO (1,2,3,4),ILOAD
    1 XFNX=XNX
      GO TO 5
    2 XFNP=XNX
      DO 6 I1=1,NEQPOT
    6 AXPRES(I1,1)=XFNP*XXPRES(I1,1)
      GO TO 5
    3 XFNY=XNX
      GO TO 5
    4 XFNP=XNX
      DO 7 I1=1,NEQPOT
    7 AXPRES(I1,1)=XFNP*XXPRES(I1,1)
      XFNX=FNI*AXPRES(1,1)
    5 CONTINUE
      RETURN
      END
```

```
      FUNCTION ALB(I,J,L,B,JP,N2,N3,N4,LL)
C   N4=1 FOR  B(JP,I+J)   OR  B(JP,I-J)
C     N4=2  FOR  B(JP,J)
C   LL=1  FOR A     LL=2 FOR  B,C,D
      COMMON/FOUR1/KFOUR,K1,K2,K3,K4,K5,K6,K7,K8,K9
      COMMON/FOUR2/K10,K11,K12,K13,K14,K15,KK2
      DIMENSION  B(N2,N3)
      IF(L.GT.3) GO TO 10
      I1=I+J
      I2=I1
      GO TO 20
   10 I1=IABS(I-J)
      I2=I1
   20 IF(N4.EQ.1) GO TO 120
      I2=J
      GO TO 100
  120 IF(I1.LE.KFOUR) GO TO 100
      ALB=0.
      RETURN
  100 IF(L.LE.3) GO TO 110
      ETA=1.
      IF(I.EQ.J) ETA=0.
  110 GO TO (11,12,13,14,15,16,22,23,24),L
   11 R1=I1**2
      GO TO 17
   12 R1=J**2
      GO TO 17
   13 R1=2.*I1*J
      GO TO 17
   14 R1=(2.-ETA)*I1**2
      GO TO 17
   15 R1=(2.-ETA)*J**2
      GO TO 17
   16 IF(I-J.LT.0)ETA=-1.
      R1=-2.*ETA*J*I1
      GO TO 17
   22 IF(I-J.LT.0)ETA=-1.
      R1=-ETA*I1**2
      GO TO 17
   23 IF(I-J.LT.0)ETA=-1.
      R1=-ETA*J**2
      GO TO 17
   24 R1=2.*(2.-ETA)*I1*J
   17 IF(LL.EQ.1)I2=I2+1
      ALB=R1*B(JP,I2)
      RETURN
      END
```

```
SUBROUTINE COEFNN(NNN)
COMMON/GEOM/RR,DD,H11,H12,H22,Q11,Q12,Q22,D11,D12,D22
COMMON/FACTOR/C1,C2,C3,C4,C5,C6,C7,C8,C9,C10,C11,C12
 C1=NNN/RR
C2=C1**2
C3=C2**2
C4=C2*Q11/D11
C5=C2/(RR*D11)
C6=2.*DD*H12*C2
C7=2.*Q12*C2
C8=DD*H22*C3
C9=C3/(4.*D11)
C10=Q22*C3
C11=2.*D12*C2
C12=D22*C3
RETURN
END
```

```
SUBROUTINE COEFF(EX,EY,XLAMD,YLAMD,RHOX,RHOY,ELAS)
COMMON/FOUR1/KFOUR,K1,K2,K3,K4,K5,K6,K7,K8,K9
COMMON/FOUR2/K10,K11,K12,K13,K14,K15,KK2
COMMON/GEOM/RR,DD,H11,H12,H22,Q11,Q12,Q22,D11,D12,D22
COMMON/FACT2/DL1,DL2,DL3,DL4,DA1,DA2,DA3,DA4,DB2,DB3,DB4,XNI,EXXP
COMMON/CINIG/NEQPOT,MI(500)
COMMON/FIDFR/DELTA,AL1,GA1,AL2,BT2,GA2
COMMON/FACT3/XL,XH
KK2=2.*KFOUR
K1=KFOUR+1
K2=K1+1
K3=2.*KFOUR+1
K4=K3+1
K5=4*KFOUR+1
K6=K5+1
K7=6*KFOUR+1
K8=K7+1
K9=7*KFOUR+2
K10=K9+1
K11=8*KFOUR+2
K12=K11+1
K13=10*KFOUR+2
K14=K13+1
K15=12*KFOUR+2
XN2=XNI**2
XH2=XH**2
DALFA=(1.+XLAMD)*(1.+YLAMD)-XN2
DD=ELAS*XH**3/(12.*(1.-XN2))
EXXP=ELAS*XH/(1.-XN2)
H11=1.+RHOX+12.*EX**2*XLAMD*(1.+YLAMD-XN2)/(XH2*DALFA)
H22=1.+RHOY+12.*EY**2*YLAMD*(1.+XLAMD-XN2)/(XH2*DALFA)
H12=1.+12.*XNI*EX*EY*XLAMD*YLAMD/(XH2*DALFA)
Q11=-XNI*EX*XLAMD/DALFA
Q22=-XNI*EY*YLAMD/DALFA
Q12=((1.+YLAMD)*EX*XLAMD+(1.+XLAMD)*EY*YLAMD)/(2.*DALFA)
D11=(1.+XLAMD)/(DALFA*EXXP)
D12=((1.+XLAMD)*(1.+YLAMD)-XNI)/(DALFA*(1.-XNI)*EXXP)
D22=(1.+YLAMD)/(DALFA*EXXP)
DL1=DD*H11
DL2=DD*XNI*(1.+(EX*EY*XLAMD*YLAMD*12.)/(XH2*DALFA))
DL3=-EX*XLAMD*(1.+YLAMD)/DALFA
DL4=XNI*EX*XLAMD/DALFA
DA1=(1.+YLAMD)/(DALFA*EXXP)
DA2=-XNI/(DALFA*EXXP)
DA3=(1.+YLAMD)*EX*XLAMD/DALFA
DA4=-XNI*EY*YLAMD/DALFA
DB2=(1.+XLAMD)/(DALFA*EXXP)
DB3=-XNI*EX*XLAMD/DALFA
DB4=(1.+XLAMD)*EY*YLAMD/DALFA
MI(1)=2*K15
MI(NEQPOT)=2*K15
NEQ1=NEQPOT-1
DO 10 I1=2,NEQ1
```

```
      MI(I1)=K15
   10 CONTINUE
      DELTA=XL/(NEQPOT-1)
      AL1=-1./(2.*DELTA)
      GA1=1./(2.*DELTA)
      AL2=1./(DELTA**2)
      BT2=-2./DELTA**2
      GA2=1./DELTA**2
      RETURN
      END
```

```
      SUBROUTINE  BOUNDR(BS,BT,BG,IN,XNXX,XNXY,LS,M1,NJ,NAW,NBW,NF,
     1LN,NRHS)
      COMMON/FOUR1/KFOUR,K1,K2,K3,K4,K5,K6,K7,K8,K9
      COMMON/FOUR2/K10,K11,K12,K13,K14,K15,KK2
      COMMON/GEOM/RR,DD,H11,H12,H22,Q11,Q12,Q22,D11,D12,D22
      COMMON/FACTOR/C1,C2,C3,C4,C5,C6,C7,C8,C9,C10,C11,C12
      COMMON/FACT2/DL1,DL2,DL3,DL4,DA1,DA2,DA3,DA4,DB2,DB3,DB4,XNI,EXXP
      COMMON/PRES1/AWM(100,3),AWMP(100,3),AWMPP(100,3)
      COMMON/PRES2/BWM(100,2),BWMP(100,2),BWMPP(100,2)
      COMMON/PRES3/CFM(100,4),CFMP(100,4),CFMPP(100,4)
      COMMON/PRES4/DFM(100,4),DFMP(100,4),DFMPP(100,4)
      COMMON/PRES5/AWZ(100,3),AWZP(100,3),AWZPP(100,3)
      COMMON/PRES6/BWZ(100,2),BWZP(100,2),BWZPP(100,2)
      DIMENSION BS(M1,M1),BT(M1,M1),BG(M1,NRHS)
      AL=1.
      IF(LN.EQ.1)AL=0.
      IF(LS.EQ.11)LS=3
      DO 1 I1=1,K15
      BG(I1,NRHS)=0.
      DO 1 J1=1,K15
      BS(I1,J1)=0.
      BT(I1,J1)=0.
    1 CONTINUE
      IF(LS.NE.10) GO TO 100
      DO 2 I1=1,K3
    2 BS(I1,I1)=1.
      BS(K8,K3)=DL4*Q11/D11-DL1
      BG(K8,NRHS)=XNXX*AWZP(IN,1)
      E4=-C2/2.*DL4/D11
      E5=-C2/2.
      DO 3 J=1,KFOUR
      JS=J**2
      J1=J+1
      BT(K8,J1)=E4*JS*AWZP(IN,J1)
      BT(K8,K1+J)=E4*JS*BWZP(IN,J)
      BT(K8,K3+J)=E5*JS*AWZP(IN,J1)
      BT(K8,K5+J)=E5*JS*BWZP(IN,J)
    3 CONTINUE
      K81=K8+1
      I=0
      DO 4 I1=K81,K9
      I=I+1
      BS(I1,K8+I)=-DL1
      BS(I1,K11+I)=-DL4
      BT(I1,K1+I)=XNXY*C1*I
      BG(I1,NRHS)=XNXX*AWZP(IN,I+1)-XNXY*BWZ(IN,I)*C1*I
      DO 5 J=1,KK2
      BT(I1,K3+J)=BT(I1,K3+J)+E5*(ALB(I,J,2,AWZP,IN,NJ,NAW,1,1)+
     1ALB(I,J,5,AWZP,IN,NJ,NAW,1,1))
      BT(I1,K5+J)=BT(I1,K5+J)+E5*(ALB(I,J,2,BWZP,IN,NJ,NBW,1,2)+
     1ALB(I,J,3,BWZP,IN,NJ,NBW,1,2))
    5 CONTINUE
    4 CONTINUE
```

```
      I=0
      DO 6 I1=K10,K11
      I=I+1
      BS(I1,K9+I)=-DL1
      BS(I1,K13+I)=-DL4
      BT(I1,I+1)=-XNXY*C1*I
      BG(I1,NRHS)=XNXX*BWZP(IN,I)+XNXY*C1*I*AWZ(IN,I+1)
      DO 7 J=1,KK2
      BT(I1,K3+J)=BT(I1,K3+J)+E5*(ALB(I,J,2,BWZP,IN,NJ,NBW,1,2)-
     1ALB(I,J,8,BWZP,IN,NJ,NBW,1,2))
      BT(I1,K5+J)=BT(I1,K5+J)+E5*(-ALB(I,J,2,AWZP,IN,NJ,NAW,1,1)+
     1ALB(I,J,5,AWZP,IN,NJ,NAW,1,1))
    7 CONTINUE
    6 CONTINUE
      DO 8 I1=K4,K7
    8 BS(I1,I1)=1.
      DO 9 I=1,KK2
      BS(K11+I,K11+I)=DB2
      BS(K13+I,K13+I)=DB2
      IF(I.GT.KFOUR) GO TO 55
      BS(K11+I,K8+I)=DB3
      BS(K13+I,K9+I)=DB3
   55 DO 10 J=1,KFOUR
      BT(K11+I,J+1)=BT(K11+I,J+1)+E5*(ALB(I,J,2,AWZP,IN,NJ,NAW,1,1)+
     1ALB(I,J,5,AWZP,IN,NJ,NAW,1,1))
      BT(K11+I,K1+J)=BT(K11+I,K1+J)+E5*(ALB(I,J,2,BWZP,IN,NJ,NBW,1,2)+
     1ALB(I,J,8,BWZP,IN,NJ,NBW,1,2))
      BT(K13+I,J+1)=BT(K13+I,J+1)+E5*(ALB(I,J,2,BWZP,IN,NJ,NBW,1,2)-
     1ALB(I,J,8,BWZP,IN,NJ,NBW,1,2))
      BT(K13+I,K1+J)=BT(K13+I,K1+J)+E5*(-ALB(I,J,2,AWZP,IN,NJ,NAW,1,1)+
     1ALB(I,J,5,AWZP,IN,NJ,NAW,1,1))
   10 CONTINUE
    9 CONTINUE
      RETURN
  100 IF(LS.NE.9) GO TO 200
      E4=DL1-DL4*D11/D11
      E5=DL4/(RR*D11)
      E6=DL4*C2/(4.*D11)
      BT(1,K8)=E4
      BS(K8,K8)=E4
      BT(1,1)=-E5
      BS(K8,1)=-E5+XNXX
      BG(K8,NRHS)=-XNXX*AWZP(IN,1)
      DO 11 J=1,KFOUR
      JS=J**2
       J1=J+1
      BT(1,J1)=BT(1,J1)+2.*E6*JS*(AL*AWM(IN,J1)+AWZ(IN,J1))
      BT(1,K1+J)=BT(1,K1+J)+2.*E6*JS*(AL*BWM(IN,J)+BWZ(IN,J))
      BG(1,NRHS)=BG(1,NRHS)+E6*JS*AL*(AWM(IN,J1)**2+BWM(IN,J)**2)
      BS(K8,J1)=BS(K8,J1)+2.*E6*JS*(AL*AWM(IN,J1)+AWZ(IN,J1))
      BT(K8,J1)=BT(K8,J1)+2.*E6*JS*(AL*AWMP(IN,J1)+AWZP(IN,J1))
      BS(K8,K1+J)=BS(K8,K1+J)+2.*E6*JS*(AL*BWM(IN,J)+BWZ(IN,J))
      BT(K8,K1+J)=BT(K8,K1+J)+2.*E6*JS*(AL*BWMP(IN,J)+BWZP(IN,J))
```

```
      BG(K8,NRHS)=BG(K8,NRHS)+2.*E6*JS*AL*(AWMP(IN,J1)*AWM(IN,J1)+
     1BWMP(IN,J)*BWM(IN,J))
   11 CONTINUE
      DO 12 I=1,KFOUR
      I1=I+1
      BT(I1,K8+I)=DL1
      BT(I1,I1)=-DL2*C2*I**2
      BT(I1,K11+I)=DL4
      BT(K1+I,K9+I)=DL1
      BT(K1+I,K1+I)=-DL2*C2*I**2
      BT(K1+I,K13+I)=DL4
      BS(K8+I,K8+I)=DL1
      BS(K8+I,I1)=-C2*I**2*(DL2+2.*DD*(1.-XNI))+XNXX
      BS(K8+I,K11+I)=DL4
      BT(K8+I,K1+I)=-XNXY*I*C1
      BG(K8+I,NRHS)=-XNXX*AWZP(IN,I1)+XNXY*I*C1*BWZ(IN,I)
      BS(K9+I,K9+I)=DL1
      BS(K9+I,K1+I)=-C2*I**2*(DL2+2.*DD*(1.-XNI))+XNXX
      BS(K9+I,K13+I)=DL4
      BT(K9+I,I1)=XNXY*C1*I
      BG(K9+I,NRHS)=-XNXX*BWZP(IN,I)-XNXY*C1*I*AWZ(IN,I1)
   12 CONTINUE
      DO 13 I=1,KK2
      BT(K3+I,K3+I)=1.
      BS(K11+I,K3+I)=1.
      BT(K5+I,K5+I)=1.
      BS(K13+I,K5+I)=1.
   13 CONTINUE
      RETURN
  200 DO 14 I1=1,K3
   14 BT(I1,I1)=1.
      IF(LS.LE.4) GO TO 300
      J=0
      DO 15 I1=K8,K11
      J=J+1
   15 BS(I1,J)=1.
      GO TO 400
  300 BT(K8,K8)=1.
      DO 16 I=1,KFOUR
      I1=I+1
      BT(K8+I,K8+I)=DL1
      BT(K8+I,K11+I)=DL4
      BT(K9+I,K9+I)=DL1
      BT(K9+I,K13+I)=DL4
      IF(LS.EQ.1.OR.LS.EQ.3) GO TO 16
      BT(K8+I,K3+I)=-DL3*I**2*C2
      BT(K9+I,K5+I)=-DL3*I**2*C2
   16 CONTINUE
  400 E4=-C2/2.
      DO 40 I=1,KK2
      IS=I**2
      I1=I+1
      GO TO(21,22,23,24,25,26,27,28),LS
```

```
21 BS(K3+I,K3+I)=1.
   BS(K5+I,K5+I)=1.
   BT(K11+I,K3+I)=1.
   BT(K13+I,K5+I)=1.
   GO TO 40
22 BS(K3+I,K3+I)=1.
   BS(K5+I,K5+I)=1.
   BS(K11+I,K11+I)=DB2
   BS(K13+I,K13+I)=DB2
   IF(I.GT.KFOUR) GO TO 41
   BS(K11+I,K8+I)=DB3
   BS(K13+I,K9+I)=DB3
   BS(K11+I,I+1)=-DB4*IS*C2+1./RR
   BS(K13+I,K1+I)=-DB4*IS*C2+1./RR
   BS(K11+I,1)=2.*E4*IS*AWZ(IN,I1)+BS(K11+I,1)
   BS(K13+I,1)=BS(K13+I,1)+2.*E4*IS*BWZ(IN,I)
41 DO 42 J=1,KFOUR
   J1=J+1
   BS(K11+I,J1)=BS(K11+I,J1)+E4*(ALB(I,J,1,AWZ,IN,NJ,NAW,1,1)+
  1ALB(I,J,4,AWZ,IN,NJ,NAW,1,1))
   BS(K11+I,K1+J)=BS(K11+I,K1+J)+E4*(ALB(I,J,1,BWZ,IN,NJ,NBW,1,2)+
  1ALB(I,J,7,BWZ,IN,NJ,NBW,1,2))
   BS(K13+I,K1+J)=BS(K13+I,K1+J)+E4*(-ALB(I,J,1,AWZ,IN,NJ,NAW,1,1)+
  1ALB(I,J,4,AWZ,IN,NJ,NAW,1,1))
   BS(K13+I,J1)=BS(K13+I,J1)+E4*(ALB(I,J,1,BWZ,IN,NJ,NBW,1,2)-
  1ALB(I,J,7,BWZ,IN,NJ,NBW,1,2))
42 CONTINUE
   GO TO 40
23 BT(K3+I,K11+I)=DB2
   BT(K5+I,K13+I)=DB2
   IF(I.GT.KFOUR) GO TO 43
   BT(K3+I,K8+I)=DB3
    BT(K5+I,K9+I)=DB3
43 BT(K11+I,K3+I)=1.
   BT(K13+I,K5+I)=1.
   GO TO 40
24 BT(K3+I,K11+I)=DB2
   BT(K5+I,K13+I)=DB2
   BT(K3+I,K3+I)=-DA2*IS*C2
   BT(K5+I,K5+I)=-DA2*IS*C2
   BS(K11+I,K3+I)=-IS*C2*(DA2+2./((1.-XNI)*EXXP))
   BS(K11+I,K11+I)=DB2
   BS(K13+I,K5+I)=-IS*C2*(DA2+2./((1.-XNI)*EXXP))
   BS(K13+I,K13+I)=DB2
   IF(I.GT.KFOUR) GO TO 44
   BT(K3+I,K8+I)=DB3
   BT(K5+I,K9+I)=DB3
   BS(K11+I,K8+I)=DB3
   BS(K11+I,I1)=-DB4*IS*C2+1./RR
   BS(K13+I,K9+I)=DB3
   BS(K13+I,K1+I)=-DB4*IS*C2+1./RR
   BS(K11+I,1)=BS(K11+I,1)+2.*E4*IS*AWZ(IN,I1)
   BS(K13+I,1)=BS(K13+I,1)+2.*E4*IS*BWZ(IN,I)
```

```
44 DO 45 J=1,KFOUR
   J1=J+1
   BS(K11+I,J1)=BS(K11+I,J1)+E4*(ALB(I,J,1,AWZ,IN,NJ,NAW,1,1)+
  1ALB(I,J,4,AWZ,IN,NJ,NAW,1,1))
   BS(K11+I,K1+J)=BS(K11+I,K1+J)+E4*(ALB(I,J,1,BWZ,IN,NJ,NBW,1,2)+
  1ALB(I,J,7,BWZ,IN,NJ,NBW,1,2))
   BS(K13+I,K1+J)=BS(K13+I,K1+J)+E4*(-ALB(I,J,1,AWZ,IN,NJ,NAW,1,1)+
  1ALB(I,J,4,AWZ,IN,NJ,NAW,1,1))
   BS(K13+I,J1)=BS(K13+I,J1)+E4*(ALB(I,J,1,BWZ,IN,NJ,NBW,1,2)-
  1ALB(I,J,7,BWZ,IN,NJ,NBW,1,2))
45 CONTINUE
   GO TO 40
25 BS(K3+I,K3+I)=1.
   BS(K5+I,K5+I)=1.
   BT(K11+I,K3+I)=1.
   BT(K13+I,K5+I)=1.
   GO TO 40
26 BS(K3+I,K3+I)=1.
   BS(K5+I,K5+I)=1.
   BS(K11+I,K11+I)=DB2
   BS(K13+I,K13+I)=DB2
   IF(I.GT.KFOUR) GO TO 40
   BS(K11+I,K8+I)=DB3
   BS(K13+I,K9+I)=DB3
   GO TO 40
27 BT(K3+I,K3+I)=1.
   BT(K5+I,K5+I)=1.
   BT(K11+I,K11+I)=DB2
   BT(K13+I,K13+I)=DB2
   IF(I.GT.KFOUR) GO TO 40
   BT(K11+I,K8+I)=DB3
   BT(K13+I,K9+I)=DB3
   GO TO 40
28 BT(K3+I,K11+I)=DB2
   BT(K5+I,K13+I)=DB2
   BT(K3+I,K3+I)=-DA2*IS*C2
   BT(K5+I,K5+I)=-DA2*IS*C2
   BS(K11+I,K3+I)=-IS*C2*(DA2+2./((1.-XNI)*EXXP))
   BS(K11+I,K11+I)=DB2
   BS(K13+I,K5+I)=-IS*C2*(DA2+2./((1.-XNI)*EXXP))
   BS(K13+I,K13+I)=DB2
   IF(I.GT.KFOUR) GO TO 40
   BT(K3+I,K8+I)=DB3
   BT(K5+I,K9+I)=DB3
   BS(K11+I,K8+I)=DB3
   BS(K13+I,K9+I)=DB3
40 CONTINUE
   RETURN
   END
```

```
      SUBROUTINE  RSTG(R,S,T,G,JP,XNXX,XNXY,M1,NJ,NAW,NDW,NF,LN,NRHS)
      COMMON/FOUR1/KFOUR,K1,K2,K3,K4,K5,K6,K7,K8,K9
      COMMON/FOUR2/K10,K11,K12,K13,K14,K15,KK2
      COMMON/PRES1/AWM(100,3),AWMP(100,3),AWMPP(100,3)
      COMMON/PRES2/BWM(100,2),BWMP(100,2),BWMPP(100,2)
      COMMON/PRES3/CFM(100,4),CFMP(100,4),CFMPP(100,4)
      COMMON/PRES4/DFM(100,4),DFMP(100,4),DFMPP(100,4)
      COMMON/PRES5/AWZ(100,3),AWZP(100,3),AWZPP(100,3)
      COMMON/PRES6/BWZ(100,2),BWZP(100,2),BWZPP(100,2)
      COMMON/XXLOAD/AXPRES(100,3),BXPRES(100,2)
      COMMON/GEOM/RR,DD,H11,H12,H22,Q11,Q12,Q22,D11,D12,D22
      COMMON/FACTOR/C1,C2,C3,C4,C5,C6,C7,C8,C9,C10,C11,C12
      DIMENSION R(M1,M1),S(M1,M1),T(M1,M1),G(M1,NRHS)
      AL=1.
      IF(LN.EQ.1)AL=0.
      DO 1 I1=1,K15
      G(I1,NRHS)=0.
      DO 1 J1=1,K15
      R(I1,J1)=0.
      S(I1,J1)=0.
      T(I1,J1)=0.
    1 CONTINUE
      J1=0
      DO 2 I1=K8,K15
      T(I1,I1)=1.
      J1=J1+1
      R(I1,J1)=-1.
    2 CONTINUE
C     EQUILIBRIUM EQUATION  FOR I=0
      R(1,K8)=DD*H11+Q11**2/D11
      T(1,K8)=2.*Q11/(RR*D11)+XNXX
      T(1,1)=1./(RR**2*D11)
      G(1,NRHS)=-XNXX*AWZPP(JP,1)+AXPRES(JP,1)
      E4=-C4/2.
      E5=-C5/2.
      E6=C2/2.
      DO 3 J=1,KFOUR
      JS=J**2
      J1=J+1
      T(1,K8+J)=T(1,K8+J)+JS*(E4*(AL*AWM(JP,J1)+AWZ(JP,J1))
     1+E6*AL*CFM(JP,J))
      T(1,J1)=T(1,J1)+JS*(E4*(AL*AWMPP(JP,J1)+AWZPP(JP,J1))+E5*(AL*
     1AWM(JP,J1)+AWZ(JP,J1))+E6*AL*CFMPP(JP,J))
      S(1,J1)=S(1,J1)+2.*JS*(E4*(AL*AWMP(JP,J1)+AWZP(JP,J1))+E6*AL*
     1CFMP(JP,J))
      T(1,K9+J)=T(1,K9+J)+JS*(E4*(AL*BWM(JP,J)+BWZ(JP,J))+E6*AL*
     1DFM(JP,J))
      T(1,K1+J)=T(1,K1+J)+JS*(E4*(AL*BWMPP(JP,J)+BWZPP(JP,J))+E5*(AL*
     1BWM(JP,J)+BWZ(JP,J))+E6*AL*DFMPP(JP,J))
      S(1,K1+J)=S(1,K1+J)+2.*JS*(E4*(AL*BWMP(JP,J)+BWZP(JP,J))+
     1AL*E6*DFMP(JP,J))
      T(1,K11+J)=T(1,K11+J)+JS*E6*(AL*AWM(JP,J1)+AWZ(JP,J1))
      T(1,K13+J)=T(1,K13+J)=JS*E6*(AL*BWM(JP,J)+BWZ(JP,J))
```

```
      T(1,K3+J)=T(1,K3+J)+JS*E6*(AL*AWMPP(JP,J1)+AWZPP(JP,J1))
      T(1,K5+J)=T(1,K5+J)+JS*E6*(AL*BWMPP(JP,J)+BWZPP(JP,J))
      S(1,K3+J)=S(1,K3+J)+2.*JS*E6*(AL*AWMP(JP,J1)+AWZP(JP,J1))
      S(1,K5+J)=S(1,K5+J)+2.*JS*E6*(AL*BWMP(JP,J)+BWZP(JP,J))
      G(1,NRHS)=G(1,NRHS)+AL*JS*(E4*(AWM(JP,J1)*AWMPP(JP,J1)+
     1AWMP(JP,J1)**2+BWM(JP,J)*BWMPP(JP,J)+BWMP(JP,J)**2)+E5/2.*
     2(AWM(JP,J1)**2+BWM(JP,J)**2)  +E6*(AWM(JP,J1)*CFMPP(JP,J)+
     3BWM(JP,J)*DFMPP(JP,J)+2.*AWMP(JP,J1)*CFMP(JP,J)+2.*BWMP(JP,J)
     4*DFMP(JP,J)+AWMPP(JP,J1)*CFM(JP,J)+BWMPP(JP,J)*DFM(JP,J)))
    3 CONTINUE
C     EQUILIBRIUM  EQUATIONS  I=1,2,,,   KFOUR     SET  A
      DO 4 I1=2,K1
      I=I1-1
      IS=I**2
      IS2=IS**2
      R(I1,I1+K8-1)=DD*H11
      R(I1,I1+K11-1)=-Q11
      T(I1,I1+K8-1)=-C6*IS+XNXX
      T(I1,I1+K11-1)=C7*IS-1./RR
      T(I1,I1)=C8*IS2-AL*IS*(C4*AWMPP(JP,1)+C5*AWM(JP,1))
      IF(LN.EQ.1) GO TO 51
      DO 5 J=1,KFOUR
      J1=J+1
      T(I1,I1)=T(I1,I1)+AL*C9*IS*J**2*((AWM(JP,J1)+2.*AWZ(JP,J1))*
     1AWM(JP,J1)+(BWM(JP,J)+2.*BWZ(JP,J))*BWM(JP,J))
    5 CONTINUE
   51 CONTINUE
      T(I1,I1+K3-1)=-C10*IS2
      S(I1,I1+K1-1)=-2.*XNXY*C1*I
      T(I1,K8)=-C4*IS*(AL*AWM(JP,I1)+AWZ(JP,I1))
      T(I1,1)=-C5*IS*(AL*AWM(JP,I1)+AWZ(JP,I1))
      G(I1,NRHS)=-XNXX*AWZPP(JP,I1)+2.*XNXY*C1*I*BWZP(JP,I)+AXPRES(JP,
     1I1)-C4*IS*AL*AWM(JP,I1)*AWMPP(JP,1)  -C5*IS*AL*AWM(JP,I1)*AWM(JP,1)
      E5=C9*IS*AWM(JP,I1)*AL
      E4=C9*IS*(AL*AWM(JP,I1)+AWZ(JP,I1))
      DO 6 J=1,KFOUR
      J1=J+1
      JS=J**2
      IF(LN.EQ.1) GO TO 7
      G(I1,NRHS)=G(I1,NRHS)+E4*JS*(AWM(JP,J1)**2+BWM(JP,J)**2)+E5*JS*
     1((AWM(JP,J1)+2.*AWZ(JP,J1))*AWM(JP,J1)+(BWM(JP,J)+2.*BWZ(JP,J))*
     2BWM(JP,J))
    7 T(I1,J1)=T(I1,J1)+2.*E4*JS*(AL*AWM(JP,J1)+AWZ(JP,J1))
      T(I1,K1+J)=T(I1,K1+J)+2.*E4*JS*(AL*BWM(JP,J)+BWZ(JP,J))
    6 CONTINUE
      E6=C2/2.
      DO 8 J=1,KK2
      T(I1,K11+J)=T(I1,K11+J)+E6*(ALB(I,J,1,AWZ,JP,NJ,NAW,1,1)
     1+ALB(I,J,4,AWZ,JP,NJ,NAW,1,1))
      T(I1,K13+J)=T(I1,K13+J)+E6*(ALB(I,J,1,BWZ,JP,NJ,NBW,1,2)
     1+ALB(I,J,7,BWZ,JP,NJ,NBW,1,2))
      S(I1,K3+J)=S(I1,K3+J)+E6*(ALB(I,J,3,AWZP,JP,NJ,NAW,1,1)
     1+ALB(I,J,6,AWZP,JP,NJ,NAW,1,1))
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      S(I1,K5+J)=S(I1,K5+J)+E6*(ALB(I,J,3,BWZP,JP,NJ,NBW,1,2)
     1+ALB(I,J,9,BWZP,JP,NJ,NBW,1,2))
      T(I1,K3+J)=T(I1,K3+J)+E6*(ALB(I,J,2,AWZPP,JP,NJ,NAW,1,1)
     1+ALB(I,J,5,AWZPP,JP,NJ,NAW,1,1))
      T(I1,K5+J)=T(I1,K5+J)+E6*(ALB(I,J,2,BWZPP,JP,NJ,NBW,1,2)
     1+ALB(I,J,8,BWZPP,JP,NJ,NBW,1,2))
      IF(LN.EQ.1) GO TO 8
      T(I1,K11+J)=T(I1,K11+J)+E6*(ALB(I,J,1,AWM,JP,NJ,NAW,1,1)+
     1ALB(I,J,4,AWM,JP,NJ,NAW,1,1))
      T(I1,K13+J)=T(I1,K13+J)+E6*(ALB(I,J,1,BWM,JP,NJ,NBW,1,2)+
     1ALB(I,J,7,BWM,JP,NJ,NBW,1,2))
      S(I1,K3+J)=S(I1,K3+J)+E6*(ALB(I,J,3,AWMP,JP,NJ,NAW,1,1)+
     1ALB(I,J,6,AWMP,JP,NJ,NAW,1,1))
      S(I1,K5+J)=S(I1,K5+J)+E6*(ALB(I,J,3,BWMP,JP,NJ,NBW,1,2)+
     1ALB(I,J,9,BWMP,JP,NJ,NBW,1,2))
      T(I1,K3+J)=T(I1,K3+J)+E6*(ALB(I,J,2,AWMPP,JP,NJ,NAW,1,1)+
     1ALB(I,J,5,AWMPP,JP,NJ,NAW,1,1))
      T(I1,K5+J)=T(I1,K5+J)+E6*(ALB(I,J,2,BWMPP,JP,NJ,NBW,1,2)+
     1ALB(I,J,8,BWMPP,JP,NJ,NBW,1,2))
      IJ1=I+J
      IF(IJ1.GT.KFOUR) GO TO 80
      T(I1,IJ1+1)=T(I1,IJ1+1)+E6*(ALB(I,J,1,CFMPP,JP,NJ,NF,2,2))
      T(I1,IJ1+K1)=T(I1,IJ1+K1)+E6*ALB(I,J,1,DFMPP,JP,NJ,NF,2,2)
      S(I1,IJ1+1)=S(I1,IJ1+1)+E6*ALB(I,J,3,CFMP,JP,NJ,NF,2,2)
      S(I1,IJ1+K1)=S(I1,IJ1+K1)+E6*ALB(I,J,3,DFMP,JP,NJ,NF,2,2)
      T(I1,IJ1+K8)=T(I1,IJ1+K8)+E6*ALB(I,J,2,CFM,JP,NJ,NF,2,2)
      T(I1,IJ1+K9)=T(I1,IJ1+K9)+E6*ALB(I,J,2,DFM,JP,NJ,NF,2,2)
   80 IJ2=IABS(I-J)
      IF(IJ2.GT.KFOUR) GO TO 90
      T(I1,IJ2+1)=T(I1,IJ2+1)+E6*ALB(I,J,4,CFMPP,JP,NJ,NF,2,2)
      S(I1,IJ2+1)=S(I1,IJ2+1)+E6*ALB(I,J,6,CFMP,JP,NJ,NF,2,2)
      T(I1,IJ2+K8)=T(I1,IJ2+K8)+E6*ALB(I,J,5,CFM,JP,NJ,NF,2,2)
      IF(IJ2.EQ.0) GO TO 90
      T(I1,IJ2+K1)=T(I1,IJ2+K1)+E6*ALB(I,J,7,DFMPP,JP,NJ,NF,2,2)
      S(I1,IJ2+K1)=S(I1,IJ2+K1)+E6*ALB(I,J,9,DFMP,JP,NJ,NF,2,2)
      T(I1,IJ2+K9)=T(I1,IJ2+K9)+E6*ALB(I,J,8,DFM,JP,NJ,NF,2,2)
   90 G(I1,NRHS)=G(I1,NRHS)+E6*((ALB(I,J,1,AWM,JP,NJ,NAW,1,1)+
     1ALB(I,J,4,AWM,JP,NJ,NAW,1,1))*CFMPP(JP,J)+(ALB(I,J,1,BWM,JP,NJ,NBW
     2,1,2)+ALB(I,J,7,BWM,JP,NJ,NBW,1,2))*DFMPP(JP,J)+(ALB(I,J,3,AWMP
     3,JP,NJ,NAW,1,1)+ALB(I,J,6,AWMP,JP,NJ,NAW,1,1))*CFMP(JP,J)+
     4(ALB(I,J,3,BWMP,JP,NJ,NBW,1,2)+ALB(I,J,9,BWMP,JP,NJ,NBW,1,2))*
     5DFMP(JP,J)+(ALB(I,J,2,AWMPP,JP,NJ,NAW,1,1)+ALB(I,J,5,AWMPP,JP,NJ
     6,NAW,1,1))*CFM(JP,J)+(ALB(I,J,2,BWMPP,JP,NJ,NBW,1,2)
     7+ALB(I,J,8,BWMPP,JP,NJ,NBW,1,2))*DFM(JP,J))
    8 CONTINUE
    4 CONTINUE
C     EQUILIBRIUM EQUATIONS  I=1,2,,,, KFOUR   SET  B
      I=0
      DO 14 I1=K2,K3
      I=I+1
      I11=I+1
      IS=I**2
      IS2=IS**2
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      R(I1,K9+I)=DO*H11
      R(I1,K13+I)=-O11
      T(I1,K9+I)=-C6*IS+XNXX
      T(I1,K13+I)=C7*IS-1./RR
      T(I1,K5+I)=-C10*IS2
      S(I1,I11)=2.*XNXY*C1*I
      T(I1,K8)=-C4*IS*(AL*BWM(JP,I)+BWZ(JP,I))
      T(I1,1)=-C5*IS*(AL*BWM(JP,I)+BWZ(JP,I))
      T(I1,I+K1)=C8*IS2-C4*IS*AL*AWMPP(JP,1)-C5*IS*AL*AWM(JP,1)
      G(I1,NRHS)=-XNXX*BWZPP(JP,I)-2.*XNXY*C1*I*AWZP(JP,I11)+
     13XPRES(JP,I)-C4*IS*AL*BWM(JP,I)*AWMPP(JP,1)-C5*IS*AL*BWM(JP,I)*
     2AWM(JP,1)
      E4=C9*IS*(AL*BWM(JP,I)+BWZ(JP,I))
      E5=C9*IS*AL*BWM(JP,I)
      DO 15 J=1,KFOUR
      JS=J**2
      J1=J+1
      IF(LN.EQ.1) GO TO 17
      T(I1,K1+I)=T(I1,K1+I)+C9*IS*JS*((AWM(JP,J1)+2.*AWZ(JP,J1))*
     1AWM(JP,J1)+(BWM(JP,J)+2.*BWZ(JP,J))*BWM(JP,J))
      G(I1,NRHS)=G(I1,NRHS)+E4*JS*(AWM(JP,J1)**2+BWM(JP,J)**2)+
     1E5*JS*((AWM(JP,J1)+2.*AWZ(JP,J1))*AWM(JP,J1)+(BWM(JP,J)+2.*
     2BWZ(JP,J))*BWM(JP,J))
   17 T(I1,J1)=T(I1,J1)+2.*E4*JS*(AL*AWM(JP,J1)+AWZ(JP,J1))
      T(I1,K1+J)=T(I1,K1+J)+2.*E4*JS*(AL*BWM(JP,J)+BWZ(JP,J))
   15 CONTINUE
      E6=C2/2.
      DO 18 J=1,KK2
      T(I1,K11+J)=T(I1,K11+J)+E6*(ALB(I,J,1,BWZ,JP,NJ,NBW,1,2)-
     1ALB(I,J,7,BWZ,JP,NJ,NBW,1,2))
      T(I1,K13+J)=T(I1,K13+J)+E6*(-ALB(I,J,1,AWZ,JP,NJ,NAW,1,1)+
     1ALB(I,J,4,AWZ,JP,NJ,NAW,1,1))
      S(I1,K3+J)=S(I1,K3+J)+E6*(ALB(I,J,3,BWZP,JP,NJ,NBW,1,2)-
     1ALB(I,J,9,BWZP,JP,NJ,NBW,1,2))
      S(I1,K5+J)=S(I1,K5+J)+E6*(-ALB(I,J,3,AWZP,JP,NJ,NAW,1,1)+
     1ALB(I,J,6,AWZP,JP,NJ,NAW,1,1))
      T(I1,K3+J)=T(I1,K3+J)+E6*(ALB(I,J,2,BWZPP,JP,NJ,NBW,1,2)-
     1ALB(I,J,8,BWZPP,JP,NJ,NBW,1,2))
      T(I1,K5+J)=T(I1,K5+J)+E6*(-ALB(I,J,2,AWZPP,JP,NJ,NAW,1,1)+
     1 ALB(I,J,5,AWZPP,JP,NJ,NAW,1,1))
      IF(LN.EQ.1) GO TO 18
      T(I1,K11+J)=T(I1,K11+J)+E6*(ALB(I,J,1,BWM,JP,NJ,NBW,1,2)-
     1ALB(I,J,7,BWM,JP,NJ,NBW,1,2))
      T(I1,K13+J)=T(I1,K13+J)+E6*(-ALB(I,J,1,AWM,JP,NJ,NAW,1,1)+
     1ALB(I,J,4,AWM,JP,NJ,NAW,1,1))
      S(I1,K3+J)=S(I1,K3+J)+E6*(ALB(I,J,3,BWMP,JP,NJ,NBW,1,2)-
     1ALB(I,J,9,BWMP,JP,NJ,NBW,1,2))
      S(I1,K5+J)=S(I1,K5+J)+E6*(-ALB(I,J,3,AWMP,JP,NJ,NAW,1,1)+
     1ALB(I,J,6,AWMP,JP,NJ,NAW,1,1))
      T(I1,K3+J)=T(I1,K3+J)+E6*(ALB(I,J,2,BWMPP,JP,NJ,NBW,1,2)-
     1ALB(I,J,8,BWMPP,JP,NJ,NBW,1,2))
      T(I1,K5+J)=T(I1,K5+J)+E6*(-ALB(I,J,2,AWMPP,JP,NJ,NAW,1,1)+
     1  ALB(I,J,5,AWMPP,JP,NJ,NAW,1,1))
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      IJ1=I+J
      IF(IJ1.GT.KFOUR) GO TO 81
      T(I1,IJ1+K1)=T(I1,IJ1+K1)+E6*ALB(I,J,1,CFMPP,JP,NJ,NF,2,2)
      T(I1,IJ1+1)=T(I1,IJ1+1)-E6*ALB(I,J,1,DFMPP,JP,NJ,NF,2,2)
      S(I1,IJ1+K1)=S(I1,IJ1+K1)+E6*ALB(I,J,3,CFMP,JP,NJ,NF,2,2)
      S(I1,IJ1+1)=S(I1,IJ1+1)-E6*ALB(I,J,3,DFMP,JP,NJ,NF,2,2)
      T(I1,IJ1+K9)=T(I1,IJ1+K9)+E6*ALB(I,J,2,CFM,JP,NJ,NF,2,2)
      T(I1,IJ1+K8)=T(I1,IJ1+K8)-E6*ALB(I,J,2,DFM,JP,NJ,NF,2,2)
   81 IJ2=IABS(I-J)
      IF(IJ2.GT.KFOUR) GO TO 91
      T(I1,IJ2+1)=T(I1,IJ2+1)+E6*ALB(I,J,4,DFMPP,JP,NJ,NF,2,2)
      S(I1,IJ2+1)=S(I1,IJ2+1)+E6*ALB(I,J,6,DFMP,JP,NJ,NF,2,2)
      T(I1,IJ2+K8)=T(I1,IJ2+K8)+E6*ALB(I,J,5,DFM,JP,NJ,NF,2,2)
      IF(IJ2.EQ.0) GO TO 91
      T(I1,IJ2+K1)=T(I1,IJ2+K1)-E6*ALB(I,J,7,CFMPP,JP,NJ,NF,2,2)
      S(I1,IJ2+K1)=S(I1,IJ2+K1)-E6*ALB(I,J,9,CFMP,JP,NJ,NF,2,2)
      T(I1,IJ2+K9)=T(I1,IJ2+K9)-E6*ALB(I,J,8,CFM,JP,NJ,NF,2,2)
   91 G(I1,NRHS)=G(I1,NRHS)+E6*((ALB(I,J,1,BWM,JP,NJ,NBW,1,2)-
     1ALB(I,J,7,BWM,JP,NJ,NBW,1,2))*CFMPP(JP,J)
     2+(-ALB(I,J,1,AWM,JP,NJ,NAW,1,1)+ALB(I,J,4,AWM,JP,NJ,NAW,1,1))*DFM
     4PP(JP,J)+(ALB(I,J,3,BWMP,JP,NJ,NBW,1,2)-ALB(I,J,9,BWMP,JP,NJ,NBW,
     51,2))*CFMP(JP,J)+(-ALB(I,J,3,AWMP,JP,NJ,NAW,1,1)+ALB(I,J,6,AWMP,
     6JP,NJ,NAW,1,1))*DFMP(JP,J)+(ALB(I,J,2,BWMPP,JP,NJ,NBW,1,2)-
     7ALB(I,J,8,BWMPP,JP,NJ,NBW,1,2))*CFM(JP,J)+(-ALB(I,J,2,AWMPP,
     8JP,NJ,NAW,1,1)+ALB(I,J,5,AWMPP,JP,NJ,NAW,1,1))*DFM(JP,J))
   18 CONTINUE
   14 CONTINUE
C     COMPATIBILITY  EQUATIONS  I=1,2,,,2*KFOUR
      I=0
       DO 19 I1=K4,K5
      I=I+1
      IS=I**2
      R(I1,K11+I)=D11
      T(I1,K11+I)=-C11*IS
      T(I1,K3+I)=C12*IS**2
      IF(I.GT.KFOUR) GO TO 35
      R(I1,K8+I)=Q11
      T(I1,K8+I)=-C7*IS+1./RR
      T(I1,I+1)=C10*IS**2-C2/2.*IS*(AL*AWMPP(JP,1))
      T(I1,K8)=-C2/2.*IS*(AL*AWM(JP,I+1)+2.*AWZ(JP,I+1))
      G(I1,NRHS)=-C2/2.*IS*(AL*AWM(JP,I+1)*AWMPP(JP,1))
   35 E6=-C2/2.
      E5=E6/2.
      DO 20 J=1,KFOUR
      J1=J+1
      T(I1,K8+J)=T(I1,K8+J)+E6*(ALB(I,J,1,AWZ,JP,NJ,NAW,1,1)+
     1ALB(I,J,4,AWZ,JP,NJ,NAW,1,1))
      T(I1,K9+J)=T(I1,K9+J)+E6*(ALB(I,J,1,BWZ,JP,NJ,NBW,1,2)+
     1ALB(I,J,7,BWZ,JP,NJ,NBW,1,2))
      S(I1,J+1)=S(I1,J+1)+E6*(ALB(I,J,3,AWZP,JP,NJ,NAW,1,1)
     1+ALB(I,J,6,AWZP,JP,NJ,NAW,1,1))
      S(I1,K1+J)=S(I1,K1+J)+E6*(ALB(I,J,3,BWZP,JP,NJ,NSW,1,2)+
     1ALB(I,J,9,BWZP,JP,NJ,NBW,1,2))
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      T(I1,J+1)=T(I1,J+1)+E6*(ALB(I,J,2,AWZPP,JP,NJ,NAW,1,1)+
     1ALB(I,J,5,AWZPP,JP,NJ,NAW,1,1))
      T(I1,K1+J)=T(I1,K1+J)+E6*(ALB(I,J,2,BWZPP,JP,NJ,NBW,1,2)+
     1ALB(I,J,8,BWZPP,JP,NJ,NBW,1,2))
      IF(LN.EQ.1) GO TO 20
      T(I1,K8+J)=T(I1,K8+J)+E5*(ALB(I,J,1,AWM,JP,NJ,NAW,1,1)+
     1ALB(I,J,4,AWM,JP,NJ,NAW,1,1))
      T(I1,K9+J)=T(I1,K9+J)+E5*(ALB(I,J,1,BWM,JP,NJ,NBW,1,2)+
     1ALB(I,J,7,BWM,JP,NJ,NBW,1,2))
      S(I1,J+1)=S(I1,J+1)+E5*(ALB(I,J,3,AWMP,JP,NJ,NAW,1,1)+
     1ALB(I,J,6,AWMP,JP,NJ,NAW,1,1))
      S(I1,K1+J)=S(I1,K1+J)+E5*(ALB(I,J,3,BWMP,JP,NJ,NBW,1,2)+
     1ALB(I,J,9,BWMP,JP,NJ,NBW,1,2))
      T(I1,J+1)=T(I1,J+1)+E5*(ALB(I,J,2,AWMPP,JP,NJ,NAW,1,1)+
     1ALB(I,J,5,AWMPP,JP,NJ,NAW,1,1))
      T(I1,K1+J)=T(I1,K1+J)+E5*(ALB(I,J,2,BWMPP,JP,NJ,NBW,1,2)+
     1ALB(I,J,8,BWMPP,JP,NJ,NBW,1,2))
      IJ1=I+J
      IF(IJ1.GT.KFOUR) GO TO 36
      T(I1,IJ1+1)=T(I1,IJ1+1)+E5*ALB(I,J,1,AWMPP,JP,NJ,NAW,2,1)
      T(I1,IJ1+K1)=T(I1,IJ1+K1)+E5*ALB(I,J,1,BWMPP,JP,NJ,NBW,2,2)
      S(I1,IJ1+1)=S(I1,IJ1+1)+E5*ALB(I,J,3,AWMP,JP,NJ,NAW,2,1)
      S(I1,IJ1+K1)=S(I1,IJ1+K1)+E5*ALB(I,J,3,BWMP,JP,NJ,NBW,2,2)
      T(I1,IJ1+K8)=T(I1,IJ1+K8)+E5*ALB(I,J,2,AWM,JP,NJ,NAW,2,1)
      T(I1,IJ1+K9)=T(I1,IJ1+K9)+E5*ALB(I,J,2,BWM,JP,NJ,NBW,2,2)
   36 IJ2=IABS(I-J)
      IF(IJ2.GT.KFOUR) GO TO 37
      T(I1,IJ2+1)=T(I1,IJ2+1)+E5*ALB(I,J,4,AWMPP,JP,NJ,NAW,2,1)
      S(I1,IJ2+1)=S(I1,IJ2+1)+E5*ALB(I,J,6,AWMP,JP,NJ,NAW,2,1)
      T(I1,IJ2+K8)=T(I1,IJ2+K8)+E5*ALB(I,J,5,AWM,JP,NJ,NAW,2,1)
      IF(IJ2.EQ.0) GO TO 37
      T(I1,IJ2+K1)=T(I1,IJ2+K1)+E5*ALB(I,J,7,BWMPP,JP,NJ,NBW,2,2)
      S(I1,IJ2+K1)=S(I1,IJ2+K1)+E5*ALB(I,J,9,BWMP,JP,NJ,NBW,2,2)
      T(I1,IJ2+K9)=T(I1,IJ2+K9)+E5*ALB(I,J,8,BWM,JP,NJ,NBW,2,2)
   37 G(I1,NRHS)=G(I1,NRHS)+E5*((ALB(I,J,1,AWM,JP,NJ,NAW,1,1)+
     1ALB(I,J,4,AWM,JP,NJ,NAW,1,1))*AWMPP(JP,J1)+(ALB(I,J,1,BWM,JP,NJ,
     2NBW,1,2)+ALB(I,J,7,BWM,JP,NJ,NBW,1,2))*BWMPP(JP,J)+(ALB(I,J,3,
     3AWMP,JP,NJ,NAW,1,1)+ALB(I,J,6,AWMP,JP,NJ,NAW,1,1))*AWMP(JP,J1)+
     3(ALB(I,J,
     43,BWMP,JP,NJ,NBW,1,2)+ALB(I,J,9,BWMP,JP,NJ,NBW,1,2))*BWMP(JP,J)+
     5(ALB(I,J,2,AWMPP,JP,NJ,NAW,1,1)+ALB(I,J,5,AWMPP,JP,NJ,NAW,1,1))*
     6AWM(JP,J1)+(ALB(I,J,2,BWMPP,JP,NJ,NBW,1,2)+ALB(I,J,8,BWMPP,JP,
     7NJ,NBW,1,2))*BWM(JP,J))
   20 CONTINUE
   19  CONTINUE
C     COMPATIBILITY  EQUATIONS  I=1,2,,,,  ,,2*KFOUR   SET  B
      I=0
      DO 21 I1=K6,K7
      I=I+1
      IS=I**2
      R(I1,K13+I)=D11
      T(I1,K13+I)=-C11*IS
      T(I1,K5+I)=C12*IS**2
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      IF(I.GT.KFOUR) GO TO 45
      R(I1,K9+I)=Q11
      T(I1,K9+I)=-C7*IS+1./RR
      T(I1,K1+I)=C10*IS**2-C2/2.*IS*(AL*AWMPP(JP,1))
      T(I1,K8)=-C2/2.*IS*(AL*BWM(JP,I)+2.*BWZ(JP,I))
      G(I1,NRHS)=-C2/2.*IS*(AL*BWM(JP,I)*AWMPP(JP,1))
   45 E6=-C2/2.
      E5=E6/2.
      DO 22 J=1,KFOUR
      J1=J+1
      T(I1,K8+J)=T(I1,K8+J)+E6*(ALB(I,J,1,BWZ,JP,NJ,NBW,1,2)-
     1ALB(I,J,7,BWZ,JP,NJ,NBW,1,2))
      T(I1,K9+J)=T(I1,K9+J)+E6*(-ALB(I,J,1,AWZ,JP,NJ,NAW,1,1)+
     1ALB(I,J,4,AWZ,JP,NJ,NAW,1,1))
      S(I1,J1)=S(I1,J1)+E6*(ALB(I,J,3,BWZP,JP,NJ,NBW,1,2)-
     1ALB(I,J,9,BWZP,JP,NJ,NBW,1,2))
      S(I1,K1+J)=S(I1,K1+J)+E6*(-ALB(I,J,3,AWZP,JP,NJ,NAW,1,1)+
     1ALB(I,J,6,AWZP,JP,NJ,NAW,1,1))
      T(I1,J1)=T(I1,J1)+E6*(ALB(I,J,2,BWZPP,JP,NJ,NBW,1,2)-
     1ALB(I,J,8,BWZPP,JP,NJ,NBW,1,2))
      T(I1,K1+J)=T(I1,K1+J)+E6*(-ALB(I,J,2,AWZPP,JP,NJ,NAW,1,1)+
     1ALB(I,J,5,AWZPP,JP,NJ,NAW,1,1))
      IF(LN.EQ.1) GO TO 22
      T(I1,K8+J)=T(I1,K8+J)+E5*(ALB(I,J,1,BWM,JP,NJ,NBW,1,2)-
     1ALB(I,J,7,BWM,JP,NJ,NBW,1,2))
      T(I1,K9+J)=T(I1,K9+J)+E5*(-ALB(I,J,1,AWM,JP,NJ,NAW,1,1)+
     1ALB(I,J,4,AWM,JP,NJ,NAW,1,1))
      S(I1,J1)=S(I1,J1)+E5*(ALB(I,J,3,BWMP,JP,NJ,NBW,1,2)-
     1ALB(I,J,9,BWMP,JP,NJ,NBW,1,2))
      S(I1,K1+J)=S(I1,K1+J)+E5*(-ALB(I,J,3,AWMP,JP,NJ,NAW,1,1)+
     1ALB(I,J,6,AWMP,JP,NJ,NAW,1,1))
      T(I1,J1)=T(I1,J1)+E5*(ALB(I,J,2,BWMPP,JP,NJ,NBW,1,2)-
     1ALB(I,J,8,BWMPP,JP,NJ,NBW,1,2))
      T(I1,K1+J)=T(I1,K1+J)+E5*(-ALB(I,J,2,AWMPP,JP,NJ,NAW,1,1)+
     1ALB(I,J,5,AWMPP,JP,NJ,NAW,1,1))
      IJ1=I+J
      IF(IJ1.GT.KFOUR) GO TO 46
      T(I1,IJ1+K1)=T(I1,IJ1+K1)+E5*ALB(I,J,1,AWMPP,JP,NJ,NAW,2,1)
      T(I1,IJ1+1)=T(I1,IJ1+1)-E5*ALB(I,J,1,BWMPP,JP,NJ,NBW,2,2)
      S(I1,IJ1+K1)=S(I1,IJ1+K1)+E5*ALB(I,J,3,AWMP,JP,NJ,NAW,2,1)
      S(I1,IJ1+1)=S(I1,IJ1+1)-E5*ALB(I,J,3,BWMP,JP,NJ,NBW,2,2)
      T(I1,IJ1+K9)=T(I1,IJ1+K9)+E5*ALB(I,J,2,AWM,JP,NJ,NAW,2,1)
      T(I1,IJ1+K8)=T(I1,IJ1+K8)-E5*ALB(I,J,2,BWM,JP,NJ,NBW,2,2)
   46 IJ2=IABS(I-J)
      IF(IJ2.GT.KFOUR) GO TO 47
      T(I1,IJ2+1)=T(I1,IJ2+1)+E5*ALB(I,J,4,BWMPP,JP,NJ,NBW,2,2)
      S(I1,IJ2+1)=S(I1,IJ2+1)+E5*ALB(I,J,6,BWMP,JP,NJ,NBW,2,2)
      T(I1,IJ2+K8)=T(I1,IJ2+K8)+E5*ALB(I,J,5,BWM,JP,NJ,NBW,2,2)
      IF(IJ2.EQ.0) GO TO 47
      T(I1,IJ2+K1)=T(I1,IJ2+K1)-E5*ALB(I,J,7,AWMPP,JP,NJ,NAW,2,1)
      S(I1,IJ2+K1)=S(I1,IJ2+K1)-E5*ALB(I,J,9,AWMP,JP,NJ,NAW,2,1)
      T(I1,IJ2+K9)=T(I1,IJ2+K9)-E5*ALB(I,J,8,AWM,JP,NJ,NAW,2,1)
   47 G(I1,NRHS)=G(I1,NRHS)+E5*((ALB(I,J,1,BWM,JP,NJ,NBW,1,2)-
```

```
     1ALB(I,J,7,BWM,JP,NJ,NBW,1,2))*AWMPP(JP,J1)+(-ALB(I,J,1,AWM,JP,
     2NJ,NAW,1,1)+ALB(I,J,4,AWM,JP,NJ,NAW,1,1))*BWMPP(JP,J)+(ALB(I,J,
     33,BWMP,JP,NJ,NBW,1,2)-ALB(I,J,9,BWMP,JP,NJ,NBW,1,2))*AWMP(JP,J1)+
     4(-ALB(I,J,3,AWMP,JP,NJ,NAW,1,1)+ALB(I,J,6,AWMP,JP,NJ,NAW,1,1))*
     5BWMP(JP,J)+(ALB(I,J,2,BWMPP,JP,NJ,NBW,1,2)-ALB(I,J,8,BWMPP,JP,
     6NJ,NBW,1,2))*AWM(JP,J1)+(-ALB(I,J,2,AWMPP,JP,NJ,NAW,1,1)+
     7ALB(I,J,5,AWMPP,JP,NJ,NAW,1,1))*BWM(JP,J))
  22 CONTINUE
  21 CONTINUE
      RETURN
     END
```

```
      SUBROUTINE POTSN(POT,STRYU,STRAU,STRYG,STRAG,XNXX,XNXY)
C STRYU   UNIT END SHORTENING  U FOR Y=0.
CSTRAU  AVERAGE UNIT END SHORTENING U
C STRYG   UNIT END SHORTENING GAMA FOR Y=C.
CSTRAG  AVERAGE END SHORTENING GAMA
      COMMON/FOUR1/KFOUR,K1,K2,K3,K4,K5,K6,K7,K8,K9
      COMMON/FOUR2/K10,K11,K12,K13,K14,K15,KK2
      COMMON/GEOM/RR,DB,H11,H12,H22,Q11,Q12,Q22,D11,D12,D22
      COMMON/FACTOR/C1,C2,C3,C4,C5,C6,C7,C8,C9,C10,C11,C12
      COMMON/FACT2/DL1,DL2,DL3,DL4,DA1,DA2,DA3,DA4,DB2,DB3,DB4,XNI,EXXP
      COMMON/CINTG/NEQPOT,MI(500)
      COMMON/FIDFR/DELTA,AL1,GA1,AL2,BT2,GA2
      COMMON/FACT3/XL,XH
      COMMON/PRES1/AWM(100,3),AWMP(100,3),AWMPP(100,3)
      COMMON/PRES2/BWM(100,2),BWMP(100,2),BWMPP(100,2)
      COMMON/PRES3/CFM(100,4),CFMP(100,4),CFMPP(100,4)
      COMMON/PRES4/DFM(100,4),DFMP(100,4),DFMPP(100,4)
      COMMON/PRES5/AWZ(100,3),AWZP(100,3),AWZPP(100,3)
      COMMON/PRES6/BWZ(100,2),BWZP(100,2),BWZPP(100,2)
      COMMON/XXLOAD/AXPRES(100,3),BXPRES(100,2)
      POT=0.
      STRYU=0.
      STRAU=0.
      STRYG=0.
      STRAG=0.
      DO 10  I1=1,NEQPOT
      E7=1.
      IF(I1.EQ.1.OR.I1.EQ.NEQPOT) E7=0.5
      E1=-Q11*AWMPP(I1,1)-AWM(I1,1)/RR
      E2=0.
      DO 11  J=1,KFOUR
      JS=J**2
      J1=J+1
      E2=E2+JS*((AWM(I1,J1)+2.*AWZ(I1,J1))*AWM(I1,J1)+(BWM(I1,J)+
     12.*BWZ(I1,J))*BWM(I1,J))
   11 CONTINUE
      E1=E1+C2/4.*E2
      PE1=1./D11*E1**2+DL1*AWMPP(I1,1)**2-2.*XNXX*DA2/D11*E1-2.*AWM(I1,
     11)*AXPRES(I1,1)
      E2=-DA2/D11*E1-DA3*AWMPP(I1,1)
      PSYU=E2
      PSAU=E2+0.5*AWMP(I1,1)*(AWMP(I1,1)+2.*AWZP(I1,1))
      E1=0.
      E2=0.
       E3=0.
      DO 18  J1=1,K1
      DO 18  J2=1,K1
      E1=E1+AWMP(I1,J1)*(AWMP(I1,J2)+2.*AWZP(I1,J2))
      IF(J1.EQ.K1) GO TO 19
      E2=E2+J1*C1*BWM(I1,J1)*AWZP(I1,J2)
   19 IF(J2.EQ.K1) GO TO 18
      E3=E3+AWMP(I1,J1)*J2*C1*(BWM(I1,J2)+BWZ(I1,J2))
   18 CONTINUE
```

```
      PSYU=PSYU+E1/2.
      PSYG=E2+E3
      E1=0.
      E2=0.
      E3=0.
      E4=0.
      E5=0.
      E6=0.
      E8=0.
      E9=0.
      E10=0.
      DO 20 J=1,KFOUR
      J1=J+1
      JS=J**2
      JS2=JS**2
      E1=E1+JS2*(AWM(I1,J1)**2+BWM(I1,J)**2)
      E2=E2+AWMPP(I1,J1)**2+BWMPP(I1,J)**2
      E3=E3+JS*(AWM(I1,J1)*AWMPP(I1,J1)+BWM(I1,J)*BWMPP(I1,J))
      E4=E4+JS*(AWMP(I1,J1)**2+BWMP(I1,J)**2)
      E5=E5+AXPRES(I1,J1)*AWM(I1,J1)+BXPRES(I1,J)*BWM(I1,J)
      E6=E6+JS*AWM(I1,J1)
      E8=E8+AWMPP(I1,J1)
      E9=E9+AWMP(I1,J1)*(AWMP(I1,J1)+2.*AWZP(I1,J1))+BWMP(I1,J)*(
     1BWMP(I1,J)+2.*BWZP(I1,J))
      E10=E10+J*(AWMP(I1,J1)*(BWM(I1,J)+BWZ(I1,J))+BWM(I1,J)*AWZP(I1,J1
     1)-BWMP(I1,J)*(AWM(I1,J1)+AWZ(I1,J1))-AWM(I1,J1)*BWZP(I1,J))
   20 CONTINUE
      PE1=PE1+C3*DD*H22/2.*E1+DL1*E2/2.-C2*DL2*E3+(1.-XNI)*DD*C2*E4-E5
      PSYU=PSYU+DA4*C2*E6-DA3*E8
      PSAU=PSAU+E9/4.
      PSAG=C1*E10
      E1=0.
      E2=0.
      E3=0.
      E4=0.
      E5=0.
      E6=0.
      E8=0.
      DO 21 J=1,KK2
      JS=J**2
      JS2=JS**2
      E1=E1+JS2*(CFM(I1,J)**2+DFM(I1,J)**2)
      E2=E2+CFMPP(I1,J)**2+DFMPP(I1,J)**2
      E3=E3+JS*(CFM(I1,J)*CFMPP(I1,J)+DFM(I1,J)*DFMPP(I1,J))
      E4=E4+JS*(CFMP(I1,J)**2+DFMP(I1,J)**2)
      E5=E5+JS*CFM(I1,J)
      E6=E6+CFMPP(I1,J)
      E8=E8+J*DFMP(I1,J)
   21 CONTINUE
      PE1=PE1+C3*D22/2.*E1+D11/2.*E2-C2*DA2*E3+C2*E4/((1.-XNI)*EXXP)
      PSYU=PSYU+C2*DA1*E5-DA2*E6
      PSYG=PSYG+C1*2./((1.-XNI)*EXXP)*E8
      POT=POT+PE1*E7
```

```
      STRYU=STRYU+PSYU*E7
      STRAU=STRAU+PSAU*E7
      STRYG=STRYG+PSYG*E7
      STRAG=STRAG+PSAG*E7
   10 CONTINUE
      STRYU=DA1*XNXX+STRYU*DELTA/XL
      STRAU=DA1*XNXX+STRAU*DELTA/XL
      STRYG=2.*XNXY/((1.-XNI)*EXXP)-STRYG*DELTA/XL
      STRAG=2.*XNXY/((1.-XNI)*EXXP)-STRAG*DELTA/(2.*XL)
      POT=3.14159*RR*(POT*DELTA+XL*(D22*XNXX**2+2./((1.-XNI)*EXXP)*
     1XNXY**2-2.*STRAU*XNXX-2.*STRAG*XNXY))
      RETURN
      END
```

```
      SUBROUTINE ABCG(IEQ,M1,CF,BF,AF,GF,NRHS,XNXX,XNXY,LN,NJ,NAW,
     1NBW,NF)
      COMMON/BOUND/LS1,LSN
      COMMON/FIDFR/DELTA,AL1,GA1,AL2,BT2,GA2
      COMMON/FOUR1/KFOUR,K1,K2,K3,K4,K5,K6,K7,K8,K9
      COMMON/FOUR2/K10,K11,K12,K13,K14,K15,KK2
      COMMON/CINTG/NEQPOT,MI(500)
      DIMENSION AF(M1,M1),BF(M1,M1),CF(M1,M1),GF(M1,NRHS)
C     LS1  KIND OF BOUNDARY CONDITION OF POINT 1
C     LSN  KIND OF BOUNDARY CONDITION OF POINT NP
C     NAW  MAXIMUM  K+1 FOR DIMENSION OF A
C      NBW  MAXIMUM  K  FOR DIMENSION OF B
C      NF  MAXIMUM  2*K  FOR DIMENSION OF C AND D
      IF(IEQ.GT.1) GO TO 10
      CALL RSTG(BF,CF,AF,GF,1,XNXX,XNXY,M1,NJ,NAW,NBW,NF,LN,NRHS)
      DO 2 I1=1,K15
      GF(I1+K15,NRHS)=GF(I1,NRHS)
      DO 2 J1=1,K15
      BF(I1+K15,J1)=AL2*BF(I1,J1)+AL1*CF(I1,J1)
      BF(I1+K15,J1+K15)=BT2*BF(I1,J1)+AF(I1,J1)
      BF(I1,J1+K15)=GA2*BF(I1,J1)+GA1*CF(I1,J1)
    2 CONTINUE
      CALL BOUNDR(AF,CF,GF,1,XNXX,XNXY,LS1,M1,NJ,NAW,NBW,NF,LN,NRHS)
      DO 3 I1=1,K15
      DO 3 J1=1,K15
      BF(I1,J1)=AL1*AF(I1,J1)
      AF(I1+K15,J1)=BF(I1,J1+K15)
      BF(I1,J1+K15)=CF(I1,J1)
      AF(I1,J1)=GA1*AF(I1,J1)
    3 CONTINUE
      RETURN
   10 IF(IEQ.GT.2) GO TO 20
      CALL RSTG(AF,CF,BF,GF,2,XNXX,XNXY,M1,NJ,NAW,NBW,NF,LN,NRHS)
      DO 4 I1=1,K15
      DO 4 J1=1,K15
      BF(I1,J1)=BF(I1,J1)+BT2*AF(I1,J1)
      CF(I1,J1+K15)=AL2*AF(I1,J1)+AL1*CF(I1,J1)
      AF(I1,J1)=GA2*AF(I1,J1)+GA1*CF(I1,J1)
      CF(I1,J1)=0.
    4 CONTINUE
      RETURN
   20 IF(IEQ.GE.NEQPOT-1) GO TO 30
      JP=IEQ
      CALL RSTG(AF,CF,BF,GF,JP,XNXX,XNXY,M1,NJ,NAW,NBW,NF,LN,NRHS)
      DO 5 I1=1,K15
      DO 5 J1=1,K15
      BF(I1,J1)=BF(I1,J1)+BT2*AF(I1,J1)
      TEMP=GA2*AF(I1,J1)+GA1*CF(I1,J1)
      CF(I1,J1)=AL2*AF(I1,J1)+AL1*CF(I1,J1)
      AF(I1,J1)=TEMP
    5 CONTINUE
      RETURN
   30 IF(IEQ.EQ.NEQPOT) GO TO 40
```

```
      JP=IEQ
      CALL RSTG(AF,CF,BF,GF,JP,XNXX,XNXY,M1,NJ,NAW,NBW,NF,LN,NRHS)
      DO 6 I1=1,K15
      DO 6 J1=1,K15
      BF(I1,J1)=BF(I1,J1)+BT2*AF(I1,J1)
      TEMP=GA2*AF(I1,J1)+GA1*CF(I1,J1)
      CF(I1,J1)=AL2*AF(I1,J1)+AL1*CF(I1,J1)
      AF(I1,J1)=TEMP
      AF(I1,J1+K15)=0.
    6 CONTINUE
      RETURN
   40 JP=IEQ
      CALL RSTG(AF,CF,BF,GF,JP,XNXX,XNXY,M1,NJ,NAW,NBW,NF,LN,NRHS)
      DO 7 I1=1,K15
      GF(I1+K15,NRHS)=GF(I1,NRHS)
      DO 7 J1=1,K15
      BF(I1,J1)=BF(I1,J1)+BT2*AF(I1,J1)
      BF(I1,J1+K15)=GA2*AF(I1,J1)+GA1*CF(I1,J1)
      BF(I1+K15,J1)=AL2*AF(I1,J1)+AL1*CF(I1,J1)
    7 CONTINUE
      CALL BOUNDR(CF,AF,GF,JP,XNXX,XNXY,LSN,M1,NJ,NAW,NBW,NF,LN,NRHS)
      DO 8 I1=1,K15
      TEMP=GF(I1,NRHS)
      GF(I1,NRHS)=GF(I1+K15,NRHS)
      GF(I1+K15,NRHS)=TEMP
      DO 8 J1=1,K15
      BF(I1+K15,J1+K15)=GA1*CF(I1,J1)
      CF(I1+K15,J1)=AL1*CF(I1,J1)
      CF(I1,J1)=BF(I1+K15,J1)
      BF(I1+K15,J1)=AF(I1,J1)
    8 CONTINUE
      RETURN
      END
```


34

```
      SUBROUTINE INVERT(NA,A,C,M,NM1,NM2,DET,IXP,IDET)
      DIMENSION A(NM1,NM1),C(NM2),M(NM2)
      DET=1.
      IXP=0
      NN=NA
      IF(NN.NE.1) GO TO 303
      DET=A(1,1)
      A(1,1)=1./A(1,1)
      GO TO 304
  303 DO 90 I=1,NN
   90 M(I)=-I
      DO 140 II=1,NN
      D=0.D0
      DO 112 K=1,NN
      IF(M(K)) 100,100,112
  100 DO 110 L=1,NN
      IF(M(L)) 103,103,110
  103 IF(ABS(D)-ABS(A(K,L)))105,105,110
  105 LD=L
      KD=K
      D=A(K,L)
      BIGA=D
  110 CONTINUE
  112 CONTINUE
      IF(D.EQ.0.D0) GO TO 170
      GO TO 188
  170 WRITE(6,502)
      STOP
  502 FORMAT(/,5X,"DETERMINANT=0"/)
  188 NEMP=-M(LD)
      M(LD)=M(KD)
      M(KD)=NEMP
      DO 114 I=1,NN
      C(I)=A(I,LD)
      A(I,LD)=A(I,KD)
  114 A(I,KD)=0.D0
      A(KD,KD)=1.D0
      DO 115 J=1,NN
  115 A(KD,J)=A(KD,J)/D
      DO 135 I=1,NN
      IF(I.EQ.KD) GO TO 135
      DO 134 J=1,NN
      TEMP=C(I)*A(KD,J)
  134 A(I,J)=A(I,J)-TEMP
  135 CONTINUE
      IF(IDET.NE.1) GO TO 140
      DET=DET*BIGA
      IF(KD.NE.LD)DET=-DET
  629 IF(ABS(DET).LT.1.E+10) GO TO 630
      DET=DET/1.E+10
      IXP=IXP+10
      GO TO 629
  630 IF(ABS(DET).GT.1.E-10) GO TO 140
```

```
      DET=DET*1.E+10
      IXP=IXP-10
140   CONTINUE
      DO 200 I=1,NN
      L=0
150   L=L+1
      IF(M(L)-I) 150,160,150
160   M(L)=M(I)
      M(I)=I
      DO 200 J=1,NN
      TEMP=A(L,J)
       A(L,J)=A(I,J)
200   A(I,J)=TEMP
304   RETURN
      END
```

```
      SUBROUTINE YMY(N1,A,B,C,N2,L1,L2,L3,T)
      DIMENSION A(L1,L2),B(L1,L1),C(L1,L2),T(L3)
      IF(N2.EQ.1) GO TO 100
      DO 11 I=1,N1
      DO 10 J=1,N2
      TEMP=0.
      DO 20 K=1,N1
   20 TEMP=TEMP+B(I,K)*C(K,J)
   10 T(J)=TEMP
      DO 30 J=1,N2
   30 A(I,J)=T(J)
   11 CONTINUE
      RETURN
  100 DO 111 I=1,N1
      TEMP=0.
      DO 120 K=1,N1
  120 TEMP=TEMP+B(I,K)*C(K,1)
  111 T(I)=TEMP
      DO 130 I=1,N1
  130 A(I,1)=T(I)
      RETURN
      END
```

```
      SUBROUTINE YSYMY(N2,N1,A,B,C,D,N3,L1,L2,L3,L4,T)
      DIMENSION A(L1,L3),B(L1,L3),C(L1,L2),D(L2,L3),T(L4)
       IF(N3.EQ.1) GO TO 100
      DO 11 I=1,N1
      DO 10 J=1,N3
      TEMP=0.
      DO 20 K=1,N2
   20 TEMP=TEMP+C(I,K)*D(K,J)
   10 T(J)=B(I,J)-TEMP
      DO 30 J=1,N3
   30 A(I,J)=T(J)
   11 CONTINUE
      RETURN
  100 DO 111 I=1,N1
      TEMP=0.
      DO 120 K=1,N2
  120 TEMP=TEMP+C(I,K)*D(K,1)
  111 T(I)=B(I,1)-TEMP
      DO 130 I=1,N1
  130 A(I,1)=T(I)
      RETURN
      END
```

```
      SUBROUTINE XREAD(ND,A,L1,L2,M1,M2,IND,M3,VV)
      COMMON/CDISK/I21(501),I22(501)
      DIMENSION A(M1,M2),VV(M3)
C     RECORD  IND OF DIRECT ACCESS DATA SET ND IS READ AND ALLOCATED
C     BY ROWS INTO L1*L2 PORTION OF MATRIX A
        L3=L1*L2
      CALL READMS(ND,VV,L3,IND)
      KL=0
      DO 10 NROW=1,L1
      DO 10 NCOL=1,L2
      KL=KL+1
      A(NROW,NCOL)=VV(KL)
   10 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE XWRITE(ND,A,L1,L2,M1,M2,IND,M3,VV)
     . COMMON/CDISK/I21(501),I22(501)
      DIMENSION A(M1,M2),VV(M3)
C   L1*L2 PORTION OF MATRIX A IS WRITTEN BY ROWS ON DIRECT ACCESS
C    DATA SET ND IN RECORD IND
      KL=0
      DO 10 NROW=1,L1
      DO 10 NCOL=1,L2
      KL=KL+1
      VV(KL)=A(NROW,NCOL)
   10 CONTINUE
      CALL WRITMS(ND,VV,KL,IND,-1)
      RETURN
      END
```


40

```
      SUBROUTINE POTERS(IDET,NRHS,MAXN,AP,BP,CP,GP,PR,XP,C,MT,T1,
     1 V1,MAX2,IXPM,DETM,XNXX,XNXY,LN,NJ,NAW,NBW,NF)
C     MAX2=MAXN*MAXN
      COMMON/CINTG/NEQPOT,MI(500)
      COMMON/CDISK/I21(501),I22(501)
      DIMENSION AP(MAXN,MAXN),BP(MAXN,MAXN),CP(MAXN,MAXN)
      DIMENSION PR(MAXN,MAXN),GP(MAXN,NRHS),XP(MAXN,NRHS)
      DIMENSION T1(MAXN),C(MAXN),MT(MAXN),V1(MAX2)
C     EQUIVALENCE (AP(1,1),V1(1))
      IXPM=0
      DETM=1.
      DO 100 I=1,NEQPOT
      CALL ABCG(I,MAXN,CP,BP,AP,GP,NRHS,XNXX,XNXY,LN,NJ,NAW,NBW,NF)
      N=MI(I)
      IF(I.EQ.1) GO TO 888
      NMIN1=MI(I-1)
  888 IF(I.EQ.NEQPOT) GO TO 999
      NPLUS1=MI(I+1)
  999 CONTINUE
      IF(I.EQ.1) GO TO 12
      CALL YSYMY(NMIN1,N,BP,BP,CP,PR,N,MAXN,MAXN,MAXN,MAXN,T1)
   12 CALL INVERT(N,BP,C,MT,MAXN,MAXN,DET,IXP,IDET)
      IF(IDET.NE.1) GO TO 640
      DETM=DET*DETM
      IXPM=IXP+IXPM
      IF(ABS(DETM).LT.1.E+10) GO TO 630
      DETM=DETM/1.E+10
      IXPM=IXPM+10
      GO TO 640
  630 IF(ABS(DETM).GT.1.E-10) GO TO 640
      DETM=DETM*1.E+10
      IXPM=IXPM-10
  640 CONTINUE
      IF(I.EQ.NEQPOT) GO TO 102
      CALL YMY(N,PR,BP,AP,NPLUS1,MAXN,MAXN,MAXN,T1)
      CALL XWRITE(21,PR,N,NPLUS1,MAXN,MAXN,I,MAX2,V1)
  102 IF(I.EQ.1) GO TO 32
      CALL YSYMY(NMIN1,N,XP,GP,CP,XP,NRHS,MAXN,MAXN,NRHS,MAXN,T1)
      CALL YMY(N,XP,BP,XP,NRHS,MAXN,NRHS,MAXN,T1)
      GO TO 42
   32 CALL YMY(N,XP,BP,GP,NRHS,MAXN,NRHS,MAXN,T1)
   42 CALL XWRITE(22,XP,N,NRHS,MAXN,NRHS,I,MAXN,T1)
  100 CONTINUE
      MEQPOT=NEQPOT-1
      DO 200 K=1,MEQPOT
      NK=NEQPOT-K
      NMIN1=MI(NK)
      N=MI(NK+1)
      CALL XREAD(21,PR,NMIN1,N,MAXN,MAXN,NK,MAX2,V1)
      CALL XREAD(22,GP,NMIN1,NRHS,MAXN,NRHS,NK,MAXN,T1)
      CALL YSYMY(N,NMIN1,XP,GP,PR,XP,NRHS,MAXN,MAXN,NRHS,MAXN,T1)
      CALL XWRITE(22,XP,NMIN1,NRHS,MAXN,NRHS,NK,MAXN,T1)
  200 CONTINUE




            RETURN
            END
```

```
      SUBROUTINE TRANSF(T1,MAXN,IDER,IPRR)
      COMMON/FOUR1/KFOUR,K1,K2,K3,K4,K5,K6,K7,K8,K9
      COMMON/FOUR2/K10,K11,K12,K13,K14,K15,KK2
      COMMON/CINTG/NEQPOT,MI(500)
      COMMON/CDISK/I21(501),I22(501)
      COMMON/FIDFR/DELTA,AL1,GA1,AL2,BT2,GA2
      COMMON/PRES1/AWM(100,3),AWMP(100,3),AWMPP(100,3)
      COMMON/PRES2/BWM(100,2),BWMP(100,2),BWMPP(100,2)
      COMMON/PRES3/CFM(100,4),CFMP(100,4),CFMPP(100,4)
      COMMON/PRES4/DFM(100,4),DFMP(100,4),DFMPP(100,4)
      COMMON/PRES5/AWZ(100,3),AWZP(100,3),AWZPP(100,3)
      COMMON/PRES6/BWZ(100,2),BWZP(100,2),BWZPP(100,2)
      COMMON/RESBN1/AWMB(2,3),AWMPPB(2,3),BWMB(2,2),BWMPPB(2,2)
      COMMON/RESBN2/CFMB(2,4),CFMPPB(2,4),DFMB(2,4),DFMPPB(2,4)
      DIMENSION T1(MAXN)
      IF(IPRR.EQ.3) GO TO 278
      DO 10 I1=1,NEQPOT
      NL=MI(I1)
      CALL READMS(22,T1,NL,I1)
      IF(I1.NE.1)GO TO 175
      DO 11 J1=1,K1
      AWMB(1,J1)=T1(J1)
      AWM(I1,J1)=T1(J1+K15)
      AWMPPB(1,J1)=T1(J1+K7)
      AWMPP(I1,J1)=T1(J1+K7+K15)
   11 CONTINUE
      DO 12 J1=1,KFOUR
      BWMB(1,J1)=T1(J1+K1)
      BWM(I1,J1)=T1(J1+K1+K15)
      BWMPPB(1,J1)=T1(J1+K9)
      BWMPP(I1,J1)=T1(J1+K9+K15)
   12 CONTINUE
      DO 13 J1=1,KK2
      CFMB(1,J1)=T1(J1+K3)
      DFMB(1,J1)=T1(J1+K5)
      CFM(I1,J1)=T1(J1+K3+K15)
      DFM(I1,J1)=T1(J1+K5+K15)
      CFMPPB(1,J1)=T1(J1+K11)
      DFMPPB(1,J1)=T1(J1+K13)
      CFMPP(I1,J1)=T1(J1+K11+K15)
      DFMPP(I1,J1)=T1(J1+K13+K15)
   13 CONTINUE
      GO TO 10
  175 DO 14 J1=1,K1
      AWM(I1,J1)=T1(J1)
      AWMPP(I1,J1)=T1(J1+K7)
   14 CONTINUE
      DO 15 J1=1,KFOUR
      BWM(I1,J1)=T1(J1+K1)
      BWMPP(I1,J1)=T1(J1+K9)
   15 CONTINUE
      DO 16 J1=1,KK2
      CFM(I1,J1)=T1(J1+K3)
```

```
      DFM(I1,J1)=T1(J1+K5)
      CFMPP(I1,J1)=T1(J1+K11)
      DFMPP(I1,J1)=T1(J1+K13)
   16 CONTINUE
      IF(I1.NE.NEQPOT)GO TO 10
      DO 17 J1=1,K1
      AWMB(2,J1)=T1(J1+K15)
      AWMPPB(2,J1)=T1(J1+K7+K15)
   17 CONTINUE
      DO 18 J1=1,KFOUR
      BWMB(2,J1)=T1(J1+K1+K15)
      BWMPPB(2,J1)=T1(J1+K9+K15)
   18 CONTINUE
      DO 19 J1=1,KK2
      CFMB(2,J1)=T1(J1+K3+K15)
      DFMB(2,J1)=T1(J1+K5+K15)
      CFMPPB(2,J1)=T1(J1+K11+K15)
      DFMPPB(2,J1)=T1(J1+K13+K15)
   19 CONTINUE
   10 CONTINUE
      IF(IDER.NE.1) GO TO 275
      NEQP=NEQPOT-1
      DO 20 I1=2,NEQP
      DO 21 J1=1,K1
      AWMP(I1,J1)=AL1*AWM(I1-1,J1)+GA1*AWM(I1+1,J1)
      IF(J1.EQ.K1) GO TO 21
      BWMP(I1,J1)=AL1*BWM(I1-1,J1)+GA1*BWM(I1+1,J1)
   21 CONTINUE
      DO 22 J1=1,KK2
      CFMP(I1,J1)=AL1*CFM(I1-1,J1)+GA1*CFM(I1+1,J1)
      DFMP(I1,J1)=AL1*DFM(I1-1,J1)+GA1*DFM(I1+1,J1)
   22 CONTINUE
   20 CONTINUE
      DO 23 J1=1,K1
      AWMP(1,J1)=AL1*AWMB(1,J1)+GA1*AWM(2,J1)
      AWMP(NEQPOT,J1)=AL1*AWM(NEQP,J1)+GA1*AWMB(2,J1)
      IF(J1.EQ.K1) GO TO 23
      BWMP(1,J1)=AL1*BWMB(1,J1)+GA1*BWM(2,J1)
      BWMP(NEQPOT,J1)=AL1*BWM(NEQP,J1)+GA1*BWMB(2,J1)
   23 CONTINUE
      DO 24 J1=1,KK2
      CFMP(1,J1)=AL1*CFMB(1,J1)+GA1*CFM(2,J1)
      DFMP(1,J1)=AL1*DFMB(1,J1)+GA1*DFM(2,J1)
      CFMP(NEQPOT,J1)=AL1*CFM(NEQP,J1)+GA1*CFMB(2,J1)
      DFMP(NEQPOT,J1)=AL1*DFM(NEQP,J1)+GA1*DFMB(2,J1)
   24 CONTINUE
  275 IF(IPRR.NE.1)RETURN
  278 CONTINUE
      J1=0
      WRITE(6,400)J1
      WRITE(6,500)
      XX=0.
      WRITE(6,600)AWMB(1,1),AWMPPB(1,1)
```

```
      DO 48 I1=1,NEQPOT
      WRITE(6,509)I1,XX,AWM(I1,1),AWMP(I1,1),AWMPP(I1,1)
      XX=XX+DELTA
   48 CONTINUE
      WRITE(6,600)AWMB(2,1),AWMPPB(2,1)
      DO 49 J1=1,KK2
      IF(J1.GT.KFOUR) GO TO 68
      WRITE(6,400)J1
      WRITE(6,500)
      WRITE(6,700)AWMB(1,J1+1),AWMPPB(1,J1+1),BWMB(1,J1),BWMPPB(1,J1)
      DO 51 I1=1,NEQPOT
      WRITE(6,609)I1,AWM(I1,J1+1),AWMP(I1,J1+1),AWMPP(I1,J1+1),
     1BWM(I1,J1),BWMP(I1,J1),BWMPP(I1,J1)
   51 CONTINUE
      WRITE(6,700)AWMB(2,J1+1),AWMPPB(2,J1+1),BWMB(2,J1),BWMPPB(2,J1)
   68 WRITE(6,400)J1
      WRITE(6,501)
      WRITE(6,700)CFMB(1,J1),CFMPPB(1,J1),DFMB(1,J1),DFMPPB(1,J1)
      DO 52 I1=1,NEQPOT
      WRITE(6,609)I1,CFM(I1,J1),CFMP(I1,J1),CFMPP(I1,J1),DFM(I1,J1),
     1DFMP(I1,J1),DFMPP(I1,J1)
   52 CONTINUE
      WRITE(6,700)CFMB(2,J1),CFMPPB(2,J1),DFMB(2,J1),DFMPPB(2,J1)
   49 CONTINUE
  400 FORMAT(//,2X,"RESULTS FOR  KFOUR=",I8/2X,"***********************
     1*************************")
  500 FORMAT(//,2X,"POINT",4X,"LENGTH",9X,"WCOS",11X,"WPCOS",10X,
     1"WPPCOS",10X,"WSIN",11X,"WPSIN",9X,"WPPSIN"/2X,"=================
     1=================================================================
     2=================================")
  501 FORMAT(//,2X,"POINT",4X,"LENGTH",9X,"FCOS",11X,"FPCOS",10X,
     1"FPPCOS",10X,"FSIN",11X,"FPSIN",9X,"FPPSIN"/2X,"==================
     1=================================================================
     2=================================")
  600 FORMAT(//20H  FICTIVE POINT       E15.6,15X,E15.6//)
  509 FORMAT(I8,E12.4,3E15.6)
  700 FORMAT(//20H  FICTIVE POINT       E15.6,15X,2E15.6,15X,E15.6//)
  609 FORMAT(I8,12X,6E15.6)
      RETURN
      END
```


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AD-A062 027 GEORGIA INST OF TECH ATLANTA SCHOOL OF ENGINEERING S--ETC F/G 20/11
THE EFFECT OF INITIAL IMPERFECTIONS ON OPTIMAL STIFFENED CYLIND--ETC(U)
JAN 78 G J SIMITSES, I SHEINMAN, J GIRI AFOSR-74-2655

UNCLASSIFIED AFOSR-TR-78-0787 NL

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      SUBROUTINE IMPERF
      COMMON/FOUR1/KFOUR,K1,K2,K3,K4,K5,K6,K7,K8,K9
      COMMON/FOUR2/K10,K11,K12,K13,K14,K15,KK2
      COMMON/CINTG/NEQPOT,MI(500)
      COMMON/FACT3/XL,XH
      COMMON/FIDFR/DELTA,AL1,GA1,AL2,BT2,GA2
      COMMON/PRES5/AWZ(100,3),AWZP(100,3),AWZPP(100,3)
      COMMON/PRES6/BWZ(100,2),BWZP(100,2),BWZPP(100,2)
      DIMENSION AM(10),BM(10)
      DV=1.
      PI=4.*ATAN(DV)
      READ(5,200)ML,DL
      READ(5,100)(AM(I),I=1,ML)
      READ(5,100)(BM(I),I=1,ML)
      DO 12 I=1,ML
      AM(I)=AM(I)*DL*XH
      BM(I)=BM(I)*XH*DL
   12 CONTINUE
  200 FORMAT(I6,E12.4)
  100 FORMAT(6E12.4)
      XX=0.
      DO 10 I1=1,NEQPOT
      AWZ(I1,2)=0.
      AWZP(I1,2)=0.
      AWZPP(I1,2)=0.
      BWZ(I1,1)=0.
      BWZP(I1,1)=0.
      BWZPP(I1,1)=0.
      AWZ(I1,1)=0.
      AWZP(I1,1)=0.
      AWZPP(I1,1)=0.
      BWZ(I1,2)=0.
      BWZP(I1,2)=0.
      BWZPP(I1,2)=0.
      DO 11 IM=1,ML
      A1=PI*IM/XL
      A12=A1**2
      A1X=SIN(A1*XX)
      A2X=COS(A1*XX)
      AWZ(I1,2)=AWZ(I1,2)-AM(IM)*A1X
      AWZP(I1,2)=AWZP(I1,2)-AM(IM)*A1*A2X
      AWZPP(I1,2)=AWZPP(I1,2)+AM(IM)*A12*A1X
      BWZ(I1,1)=BWZ(I1,1)-BM(IM)*A1X
      BWZP(I1,1)=BWZP(I1,1)-BM(IM)*A1*A2X
      BWZPP(I1,1)=BWZPP(I1,1)+BM(IM)*A12*A1X
   11 CONTINUE
      IF(K1.LE.2) GO TO 50
      DO 13 J1=3,K1
      AWZ(I1,J1)=0.
      AWZP(I1,J1)=0.
      AWZPP(I1,J1)=0.
      BWZ(I1,J1)=0.
      BWZP(I1,J1)=0.
```

```
      BWZPP(I1,J1)=0.
   13 CONTINUE
   50 CONTINUE
      XX=XX+DELTA
   10 CONTINUE
      RETURN
       END
```

APPENDIX B

Summary of Work Performed Under AFOSR-74-2655

The overall effort under grant AFOSR-74-2655 which deals with the minimum weight design of fuselage-type stiffened cylindrical shells, can best be described by giving a precise but general statement of the problem and by discussing individually all of the accomplishments.

The precise statement is as follows: given an internally stiffened, imperfect, circular, cylindrical shell of specified material, radius, and length, find the size, shape and spacings of the stiffeners and the thickness of the skin, such that the resulting configuration can safely carry a given set of destabilizing loads (applied individually or in combination) with minimum weight. The solution of the problem required application of modern optimization techniques, development of efficient solution methodologies for finding critical conditions for stiffened shells under certain loads and in the presence of initial imperfections, and incorporation of all of the above into a single computer code.

The load cases considered are (a) uniform axial compression, (b) uniform or nonuniform lateral pressure and (c) uniform torsion. These loads are representative of the types that the configuration is expected to encounter in service when used either in an aerospace vehicle fuselage or in a submarine hull.

The structural theories employed in the mathematical model are based on the following assumptions.

(1) the material behavior is linearly elastic.
(2) the kinematic relations correspond to those of moderate rotations $[(\text{rotation})^2 \ll 1]$
(3) the loads are applied quasi-statically
(4) the stiffener spacings are small and the connections are monolithic so that the "smeared" technique be applicable
(5) the imperfection shapes are smooth functions of position and the imperfection amplitude is reasonable from a manufacturing point of view (several skin thicknesses).

The overall effort can be divided in two parts (a) those problems which are free of initial imperfections and for which linear buckling analyses have been employed in the optimization technique and (b) those problems which include geometric imperfections.

The accomplishments for each class of problems are listed separately.

OPTIMIZATION OF PERFECT GEOMETRY CYLINDERS EMPLOYING LINEAR BUCKLING ANALYSES

The work associated with this phase of the research program has been reported in detail, through three AFOSR Technical Reports (Refs. B.1-B.3) and numerous publications in refereed journals (Refs. B.4-B.11).

In this phase, the design objective of the aforementioned optimization problem is minimum weight. The general instability load(s) is considered to be the one or one of the active failure modes (in some cases, applicable to pressure loaded submarine hulls, skin yielding is considered to be the active failure mode). Thus, in the optimization formulation this load is taken as an equality constraint. All other failure modes are taken as inequality constraints (behavioral). These include panel instability, local instabilities of the skin and stiffeners and yielding of the skin and stiffeners. Other inequality constraints are of the geometric type and they represent realistic dimensions for some of the design variables such as minimum gages for skin and stiffener thicknesses as well as limitations on the stiffener spacings.

Moreover, because of the possibility of detrimental effects due to failure mode interaction, the condition of separation of these modes is imposed on the solution.

The design variables of the problem are the skin thickness, the stiffener spacings, the stiffener geometries (shapes considered are: rectangular, R, angle, A, T-shapes, T, channel sections, C, I-sections, I, hat-sections, H, and others), web and flange widths and thicknesses. The number of these variables is eleven, but due to the small effect of the stiffener web and flange thicknesses on the minimum weight, it is assumed that these thicknesses are equal and thus the number is reduced to ten. The load cases considered and reported in Refs. B.1-B.11 include (i) uniform axial compression, (ii) uniform pressure, (iii) torsion, and (iv) all possible combinations of above.

Among the most important conclusions of this phase one may list: (a) the developed methodology includes the following desirable features:

(i) The designer can easily assess the need or lack of need for stiffening in both directions.
(ii) through no penalty or minimum penalty in weight the designer may avoid failure mode interaction.
(iii) the designer may carry out important trade-off studies to arrive at a practical minimum weight configuration.

(b) For axial compression and pressure loaded systems, the minimum weight design is not unique. This means that, for a given set of the specified parameters, the design variables can be adjusted so as to give several acceptable designs corresponding to the same minimum weight.

(c) The optimum distribution of material is load case dependent and length to radius ratio dependent. For moderate length cylinders ($L/R \simeq 3$), the following observations are made;

(i) The optimum distribution of material corresponds to approximately 60% in the skin, 30% in the stringers and 10% in the rings for axially loaded shells.
(ii) For pressure loaded shell, the optimum distribution of material corresponds to 60% in the skin, 30% in the rings and 10% in the

stringers.

(iii) For combined axial compression and pressure the combined stiffener material is approximately 40% of the total, but distributed into stringer and ring material in accordance with the relative amounts of pressure and axial compression (with respect to linear theory critical loads).

(iv) For pressure loaded shells the amount of ring material increases with the L/R ratio (slightly).

(d) The optimum stiffener shape is also load case dependent.

(i) For axially loaded stiffened shells the stringers must be T-shaped, while the rings must be rectangular (see Fig. B.1-B.3)

(ii) For pressure loaded stiffened shells the rings must be T-shaped, while the stringers must be rectangular.

(iii) For torsion loaded stiffened cylinders the combination of hat stringers with either hat rings or rectangular rings proved to be the most efficient one.

(e) The curve for determining the optimum skin thickness (part of the minimum weight design methodology) is relatively flat for the axial compression and uniform pressure cases. Therefore, very precise determination of the optimum thickness is not necessary for minimum weight design. In the case of torsion, the minimum weight thickness is always equal, to the minimum gage imposed.

OPTIMIZATION OF IMPERFECT STIFFENED CYLINDERS

The work associated with this phase of the program has been reported through Refs. B.12-B.17, as well as in the present AFOSR Technical Report.

In this phase a methodology was first developed and demonstrated through numerous examples for analyzing imperfect stiffened shell configurations under various load conditions (see Refs. B.12-B.15 and present report).

Then, this methodology was incorporated with the linear buckling theory optimization procedure in order to optimize an imperfect stiffened cylinder (see Refs. B.12, B.16, B.17 and present report).

Thus, a solution methodology has been developed in order to optimize an imperfect stiffened circular cylindrical shell under individual and combined application of destabilizing loads.

Finally, a small effort was exerted, under the present grant, to investigate the imperfection (load eccentricity) sensitivity of a different structural configuration (a simple two-bar frame--see Ref. B.18).

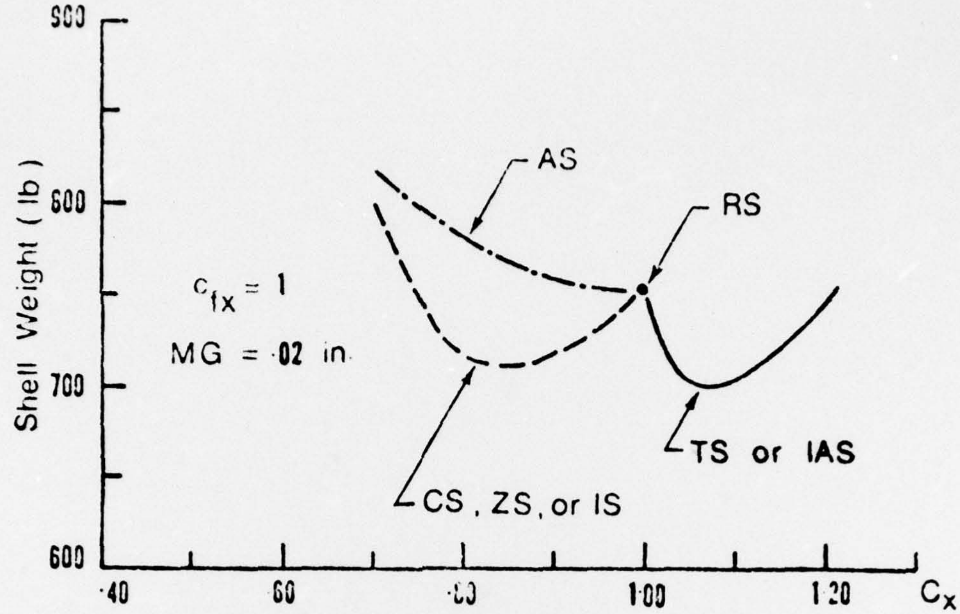

Fig B.1 Case 1. Effect of Stringer Shapes on Cylinder Weight using Rectangular Ring ($C_y = 1$).

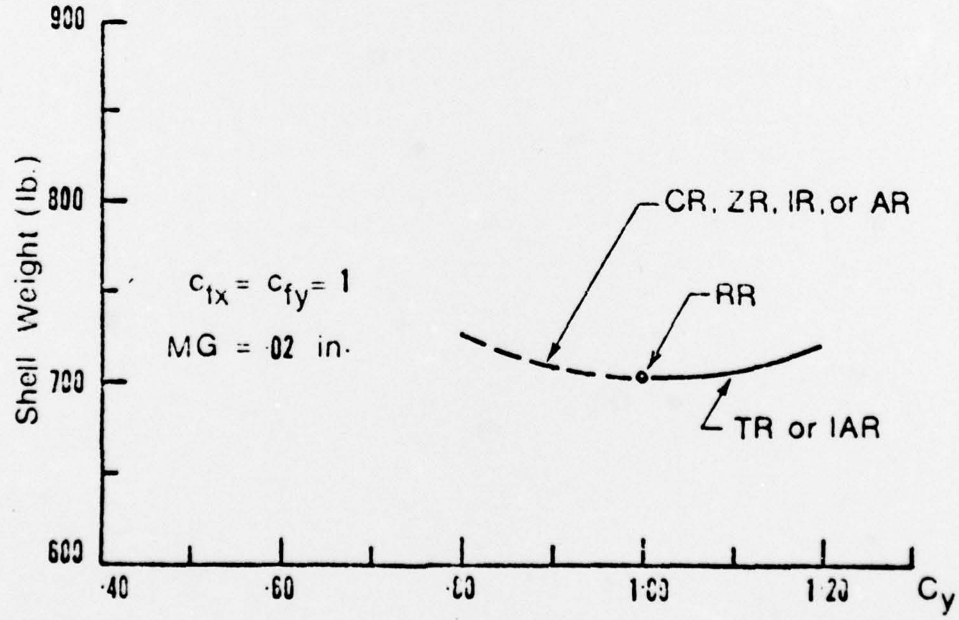

Fig B.2 Case 1. Effect of Ring Shapes on Cylinder Weight using Most Efficient Stringer (TS or IAS, C_x = 1·0[illegible]7)

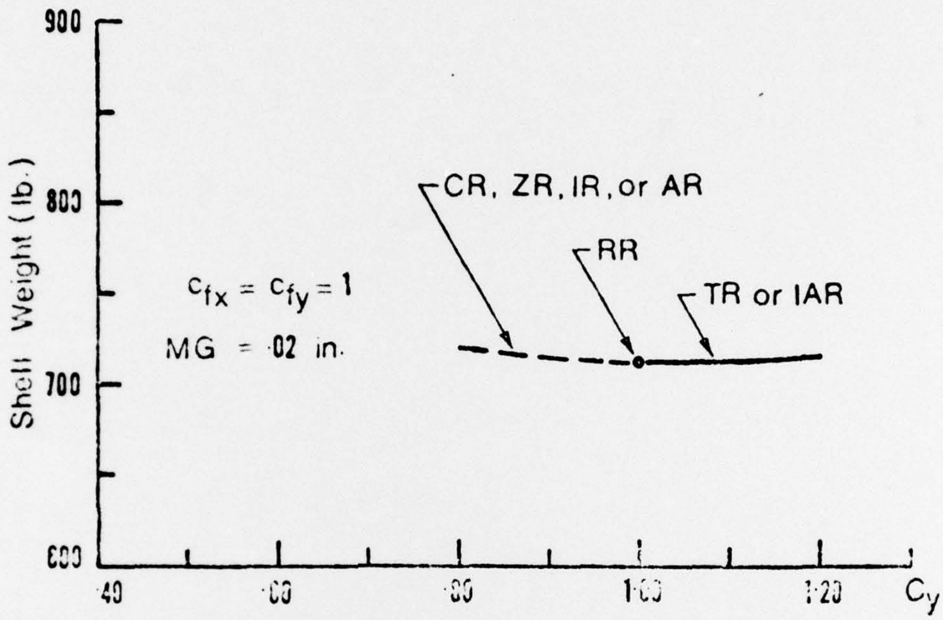

Fig.B.3 Case 1. Effect of Ring Shapes on Cylinder Weight using Most Efficient Channel Stringer (or ZS or IS, C_x = ·866).

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B.3 "Minimum Weight Design of Stiffened Cylinders and Cylindrical Panels under Combined Loads", AFOSR TR-76-0930, Georgia Tech, March 1976 (with J. Giri and I. Sheinman).

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